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M.A./M.Sc. (Semester – I) Examination, 2011
MATHEMATICS
MT-504 : Number Theory
(2008 Pattern)

Time : 3 Hours   Max. Marks : 80

N.B.: *1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) Let b and c be integers atleast one of them non-zero. Let g be the greatest common divisor of b and c. Prove that there exist integers x_0 and y_0 such that $g = (b, c) = bx_0 + cy_0$. **6**

   b) Prove that $4 \nmid (n^2 + 2)$ for any integer n. **4**

   c) Prove that no integers x, y exist satisfying x + y = 100 and (x, y) = 3. **4**

   d) Evaluate [n, n + 1] where n is a positive integer. **2**

2. a) Prove that if (a, m) = 1, then $a^{\phi(m)} \equiv 1 \pmod m$. **6**

   b) Prove that an integer is divisible by 11 if and only if the difference between the sum of the digits in odd places and the sum of the digits in the even places is divisible by 11. **5**

   c) Show that 2, 4, 6, ..., 2m is a complete residue system modulo m if m is odd. **5**

3. a) Let p denote a prime. Prove that $x^2 \equiv -1 \pmod p$ has solutions if and only if p = 2 or $p \equiv 1 \pmod 4$. **8**

   b) Prove that there exist 2011 consecutive composite integers. **4**

   c) Show that there is no x for which both $x \equiv 29 \pmod{52}$ and $x \equiv 19 \pmod{72}$. **4**

4. a) Let f(n) be a multiplicative function and let $F(n) = \sum_{d|n} f(d)$. Prove that F(n) is multiplicative. **6**

   b) Let p denote a prime. Prove that the largest exponent e such that $p^e \mid n!$ is

   $$e = \sum_{i=1}^{\infty} \left[\frac{n}{p^i} \right].$$ **5**

   c) Prove that the number of divisors of n is odd if and ond if n is a perfect square. **5**

P.T.O.

5. a) Prove that if p and q are distinct odd primes, then

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\left\{\frac{(p-1)}{2}\right\}\left\{\frac{(q-1)}{2}\right\}}.$$ **6**

b) Let p denote any odd prime. Let (a, p) = 1. Consider the integers a, 2a, 3a, ..., $\left\{(p-1)\big/z\right\}a$ and their least positive residues modulo p. If n denotes the number of these residues that exceed $p\big/2$, then prove that $\left(\frac{a}{p}\right) = (-1)^n$. **6**

c) Verify that $x^2 \equiv 10 \pmod{89}$ is solvable. **4**

6. a) Prove that the product of two primitive polynomials is primitive. **5**

b) Prove that if a monic polynomial f(x) with integral coefficients factors into two monic polynomials with rational coefficients, say f(x) = g(x) h(x), then g(x) and h(x) have integral coefficients. **6**

c) If f(x) and g(x) are primitive polynomials, and if f(x) | g(x) and g(x) | f(x), prove that $f(x) = \pm g(x)$. **5**

7. a) Prove that among the rational numbers the only ones that are algebraic integers are the integers 0, ± 1, ± 2, ... **5**

b) Find the minimal polynomial of $\sqrt{2} + \sqrt{3}$. **5**

c) Prove that the reciprocal of a unit is a unit. Also prove that the units of an algebraic number field form a multiplicative group. **6**

8. a) Prove that every Euclidean quadratic field has the unique factorization property. **8**

b) Let $Q\left(\sqrt{m}\right)$ have the unique factorization property. Prove that for any prime π in $Q\left(\sqrt{m}\right)$ there corresponds one and only one rational prime p such that π/p. **6**

c) Prove that 1 - i is a prime in Q(i). **2**

M.A./M.Sc. (Semester – II) Examination, 2011
MATHEMATICS
MT-601 : General Topology (New Course)
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B. : i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) For a non-empty set X, let $\mathfrak{I}_c$ be a collection of all subsets U of X such that X – U is either is countable or all of X. Then show that $\mathfrak{I}_c$ is a topology on X. **6**

b) Let (X, $\mathfrak{I}$) be a topological space. Suppose $\mathscr{C}$ is a collection of open sets of X such that for each open set U of X and each $x \in U \exists$ an element $C \in \mathscr{C}$ such that $x \in C \subset U$, then show that $\mathscr{C}$ is basis for the topology of X. **5**

c) Define standard topology, lower limit topology on $\mathbb{R}$ and show that lower limit topology is strictly finer than usual topology. **5**

2. a) If $\mathscr{B}$ is a basis for the topology on X and $\mathscr{C}$ is a basis for topology on Y then show that the collection $D = \{ B \times C \mid B \in \mathscr{B}, C \in \mathscr{C} \}$ is a basis for the topology of $X \times Y$. **5**

b) Let Y = [–1, 1] as a subspace of $\mathbb{R}$. Which of the following sets are open in Y ? Which are open in $\mathbb{R}$?

i) $A = \left\{ x \Big/ \frac{1}{2} < |x| \leq 1 \right\}$

ii) $B = \left\{ x \Big/ \frac{1}{2} \leq |x| < 1 \right\}$

iii) $C = \left\{ x \Big/ 0 < |x| < 1, \frac{1}{x} \notin Z_+ \right\}$. **6**

c) Let A be subset of the topological space X and A' denote the set of all limit points of A, then show that $\overline{A} = A \cup A'$. **5**

3. a) Show that X is Hausdorff space iff the diagonal $\Delta = \{x \times x / x \in X\}$ is closed in $X \times X$. **6**

b) Give an example of two discontinuous functions whose composite function is a continuous function, with proper justification. **5**

c) State and prove pasting lemma. **5**

4. a) Let $\{X_\alpha\}$ be an indexed family of topological spaces with $A_\alpha \subset X_\alpha$ for each α. If ΠX_α is given either the product or box topology then show that, $\Pi \overline{A}_\alpha = \overline{\Pi A_\alpha}$. **5**

b) Show that R^w with box topology is not metrizable. **6**

c) Define quotient topology and given an example of a quotient map which is not a closed map ? **5**

5. a) Show that the union of a collection of connected subspaces that have a common point is connected. **5**

b) Assuming that $\mathbb{R}$ is uncountable, show that if A is a countable subset of $\mathbb{R}^2$, then $R^2 - A$ is path connected. **6**

c) Show that if X is locally path connected then the components and the path components of X are the same. **5**

6. a) For locally path connected space X, show that every connected open set is path connected. **6**

b) Prove that every compact subspace of a Hausdorff space is closed. **5**

c) Show that [0, 1] is not limit point compact as a subspace of $\mathbb{R}e$. **5**

7. a) Let X be a topological space. Let one point sets in X be closed. Show that X is regular if and only if given a point $x \in X$ and a neighbourhood U of x there is a neighbourhood V of x such that $\overline{V} \subset U$. **5**

b) Show that the space $\mathbb{R}_k$ is Housdorff but not regular. **6**

c) Show that every locally compact Hausdorff space is regular. **5**

8. a) State Urysohn lemma and Urysohn Metrization theorem. **4**

b) State Tietze Extension theorem. **2**

c) State and prove Tychonoff theorem. **10**

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M.A./M.Sc. (Semester – II) Examination, 2011
MATHEMATICS
MT-601 : Real Analysis – II
(Old Course) (2005 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B. : i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) True or false ?
Bounded continuous function is of bounded variation. Justify your answer. **6**

b) Fix $f \in BV[a,b]$, and let $v(x) = V_a^x f$. Prove that f is right continuous at $x \in [a,b]$ if and only if v is right continuous at x. **6**

c) If $P \subset Q$ are partitions of [a, b], then prove that $L(f;P) \le L(f;Q)$. **4**

2. a) Suppose that f' exists and is Riemann integrable on [a, b]. Prove that

$f \in BV[a,b]$ and $V_a^b f = \int_a^b |f'(t)|\,dt.$ **6**

b) Give an example of a sequence of Riemann integrable functions on [0, 1] that converges pointwise to a non-integrable function. **6**

c) If E_1 and E_2 are measurable sets, then prove that $E_1 \cup E_2$ is measurable. **4**

3. a) Let (E_n) be a sequence of measurable sets. If $E_n \subset E_{n+1}$ for each n, then

prove that $m\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} m(E_n).$ **6**

b) Give an example of uncountable set with finite outer measure. **4**

c) Let N be a non-measurable subset of (0, 1) and let $f(x) = x \cdot \chi_N(x)$. Show that f is non-measurable, but each of the set $\{f = \alpha\}$ is measurable. **6**

4. a) If $f : [a,b] \to \mathbb{R}$ is Riemann integrable function, then prove that f is Lebesgue measurable. **6**

b) Let f be non-negative and measurable function. Prove that $\int f = 0$ if and only if f = 0 a.e. **6**

c) Show that $\lim_{n\to\infty} \int_0^1 f_n = 0$ where $f_n(x) = \dfrac{nx}{1+n^2x^2}$. **4**

5. a) If (f_n) converges uniformly to f on D, then prove that (f_n) converges a.e. and in measure to f on D. **6**

   b) State and prove Hölder’s inequality. **6**

   c) Let $f(x) = x \sin\left(\frac{1}{x}\right), \ x \neq 0, \ f(0) = 0.$ Find derived number for f at 0. **4**

6. a) If $f : [a, b] \to \mathbb{R}$ is increasing, then prove that the set of points at which at least one derived number for f is infinite has measure zero. **6**

   b) If f is increasing on [a, b], then prove that f' is measurable, $f' \geq 0$ a.e., and $\int_a^b f' \leq f(b) - f(a).$ **6**

   c) If (f_n) converges to 0 in $L_\infty(E)$, then prove that there is a null set $A \subset E$ such that (f_n) converges uniformly to 0 on E\A. **4**

7. a) Give an example where $f^2 \in R_\alpha[a, b]$ but $f \notin R_\alpha[a, b].$ **5**

   b) If G is bounded open set, then prove that for every $\varepsilon > 0$, there exists a closed set $F \subset G$ such that $m^*(F) > m^*(G) - \varepsilon.$ **5**

   c) Let $C \in \mathbb{R}$ and let $f, g : D \to \mathbb{R}$ be measurable functions. Prove that cf and f + g are measurable functions. **6**

8. a) Give an example of a function f such that f is Lebesgue integrable but f^2 is not Lebesgue integrable. **5**

   b) Prove that f is Lebesgue integrable if and only if $|f|$ is Lebesgue integrable. **6**

   c) Let $f : \mathbb{R} \to \mathbb{R}$ be $2\pi.$ Periodic and Riemann integrable on $[-\pi, \pi].$ If f is even, show that its Fourier series can be written using only cosine terms. **5**

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M.A./M.Sc. (Semester – II) Examination, 2011
MATHEMATICS
MT-603 : Groups and Rings
(2008 Patterns)

Time : 3 Hours Max. Marks : 80

***N.B.**: 1) Answer **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) If G is a finite group then prove that the number of elements x of G such that $x^3 = e$ (identity) is odd and the number of elements x of G such that $x^2 \neq e$ is even. **6**

   b) If G is a group and if it has two elements a and b such that $|a| = 4, |b| = 2$ and $a^3b = ba$, then find $|ab|$. **5**

   c) If G is the symmetry group of a circle then show that G has elements of every finite order as well as elements of infinite order. **5**

2. a) If G = <a> is a cyclic group of order n, then prove that G = $\langle a^k \rangle$ if and only if gcd (K, n) = 1. **6**

   b) If G is an Abelian group and contains cyclic subgroups of orders 4 and 5; what other sizes of cyclic subgroups must G contains ? **5**

   c) In S_4, find a cyclic subgroup of order 4 and a non-cyclic subgroup of order 4. What is the maximum order of any element in S_4 ? **5**

3. a) Prove that every group is isomorphic to a group of permutation. **6**

   b) Prove or disprove : **5**

      i) If $G \simeq H$ then Aut (G) $\simeq$ Aut (H)

      ii) If Aut G $\simeq$ Aut H then $G \simeq H$.

   c) Prove that S_3 is isomorphic to D_3 but S_4 is not isomorphic to D_{12}. **5**

4. a) i) If H and K are subgroups of a finite group G with $H \subseteq K \subseteq G$ then prove that **3**

$$|G:H| = |G:K|\,|K:H|$$

ii) Show that Q, the group of rational numbers under addition, has no proper subgroup of finite index. **3**

b) If G_1 and G_2 are two finite groups and if $(g_1, g_2) \in G_1 \oplus G_2$ then prove that

$|(g_1, g_2)| = 1\,\text{c.m}\left(|g_1|, |g_2|\right)$. **5**

c) Determine the number of cyclic subgroups of order 10 in $Z_{100} \oplus Z_{25}$. **5**

5. a) Define the centre Z(G), of a group.

Prove that for any group G, $\dfrac{G}{Z(G)}$ is isomorphic to Inn (G) where Inn (G) is the group of inner outomorphisms of G. **6**

b) If H is a subgroup of the centre Z(G), of a group G such that $\dfrac{G}{H}$ is cyclic then prove that G is Abelian. Use this to Show that Inn $(D_6) \simeq D_3$. **8**

c) If $|G| = 30$ and $|Z(G)| = 5$ then what is structure of $\dfrac{G}{Z(G)}$? **2**

6. a) If K is a subgroup of a group G and N is a normal subgroup of a group G, then prove that $\dfrac{K}{K \cap N} \simeq \dfrac{KN}{N}$. **6**

b) Determine all homomorphisms from Z_{12} to Z_{30}. **6**

c) If $\phi : U(40) \to U(40)$ is a group homomorphism with Ker $\phi = (1, 9, 17, 33)$ and $\phi(11) = 11$ then find all elements of U(40) that map to 11. **4**

7. a) Prove that if n divides the order of a finite Abelian group G then G has a subgroup of order n. **6**

b) Find all Abelian groups (up to isomorphism) of order 180. **5**

c) How many elements of order 2 are there in $Z_4 \oplus Z_4$? Use this to show that there is no homomorphism from $Z_8 \oplus Z_2 \oplus Z_2$ onto $Z_4 \oplus Z_4$. **5**

8. a) For any group G, prove that $|G| = \Sigma |G : C(a)|$ where the sum runs over one element a from each conjugacy class of G. **5**

b) Using Sylow's theorem, determine all groups of order gg. **5**

c) If G a non-cyclic group of order 21 then : **6**

i) How many Sylow 3-subgroups does G have ?

ii) Prove that G has 14 elements of order 3.

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M.A./M.Sc. (Semester – II) Examination, 2011
MATHEMATICS
MT-606 : Object Oriented Programming with C++
(2008 Pattern)

Time : 2 Hours Max. Marks : 50

*N.B. : 1) Figures at **right** indicate **full** marks.*
*2) Question **one** is **compulsory**.*
*3) Attempt **any two** questions from Q. **2**, Q. **3** and Q. **4**.*

1. Attempt the following (**2** mark **each**) (**any 10**) : **20**

1) Write syntax for accessing array element.

2) What will be output of following C++ program

```
# include <iostream.h>
void main ( )
{
  int i, j;
 for (i = 1, i < = 5, i++)
{
   cout < < endl ;
   for (j = 1; j < = i; j ++)
   cout << "*";
    }
  }
```

3) Give an example of structure in C++.

4) What will be output of following C++ program

```
# include <iostream.h>
  void main ( )
{ int a ;
  a = 555;
  cout < < "value =" < <*ptr;
 }
```

5) Interpret the following statement

int $*$ inarray [10];

6) Let 'item' be a class

declare item x;

item $*$ ptr = & x;

Is it true that $*$ ptr is alias of x ?

7) What is the task of constructer ?

8) Define hybrid inheritance.

9) Write two types of defining member function of a class.

10) What is technique to determine correct function in function overloading ?

11) What is data encapsulation ?

12) What is range of integer and float ?

2\. a) Write a C + + program to sort given array of integers in ascending order. **5**

b) Write a C++ program to read array of 10 integers in main and print it using a function prn. **5**

c) Write a note on identifiers and constants. **5**

3\. a) Define structure and write methods of declaring variable. **5**

b) Write a note on basic data types in C+ +. **5**

c) Write a note on expressions in C+ +. **5**

4\. a) What is friend function ? What are merits and demerits of using friend function ? **5**

b) Write a program to illustrate how an object can be created (within a function) and returned to another function. **5**

c) Write a note on local classes. **5**

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M.A./M.Sc. (Semester – III) Examination, 2011
MATHEMATICS (2005 Pattern) (Optional)
MT-706 : Numerical Analysis (Old)

Time : 3 Hours Max. Marks : 80

*N.B.: 1) Answer **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*
*3) Use of unprogrammable, scientific calculator is **allowed**.*

1. A) Assume that g(x) and $g^1(x)$ are continuous on a balanced interval $(a, b) = (P - \delta, P + \delta)$ that contain the unique fixed point P and that starting value P_0 is chosen in the interval. Prove that if $\left|g^1(x)\right| \le K \le 1 \forall x \in [a, b]$ then the iteration $P_n = g(P_{n-1})$ converges to P and if $\left|g^1(x)\right| > 1 \forall x \in [a, b]$ then the iteration $P_n = g(P_{n-1})$ does not converges to P. **6**

B) Investigate the nature of iteration in part (a) when $g(x) = 3(x - 2.25)^{\frac{1}{2}}$:

i) Show that P = 4.5 is only the fixed points.

ii) Use P_0 = 4.4 and compute p_1, p_2, p_3, p_4. **5**

C) Start with the interval [3.2, 4.0] and use the Bisection method to find an interval of width h = 0.05 that contain a solution of the equation log (x) – 5 + x = 0. **5**

2. A) Assume that $f \in c^2[a, b]$ and exist number $p \in [a, b]$ where f(p) = 0. If $f'(p) \ne 0$ prove that there exist a δ f 0 such that the sequence $\{p_k\}$ defined by iteration $p_k = p_{k-1} - \frac{f(p_{k-1})}{f(p_{k-1})}$. For k = 1, 2... converges to p for any initial approximation $p_0 \in [p - \delta, p + \delta]$. **6**

B) Let $f(x) = (x - 2)^4$:

i) Find Newton-Raphson formula.

ii) Start with p_0 = 2.1 and compute p_1, p_2, p_3.

iii) Is the sequence converging quadratically or linearly ? **5**

P.T.O.

C) Solve the system of equation : **5**

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

Using the Gauss Elimination Method with partial Pivoting.

3. A) Explain Gaussian Elimination method for solving a system of m equation in n knows. **6**

B) Find the Jacobian J(X, Y, Z) of order 3 3 at the point (1, 3, 2) for the functions :

$f_1(X, Y, Z) = X^3 - Y^2 + Y - Z^4$, $f_2(X, Y, Z) = XY + YZ + XZ$, $f_3(X, Y, Z) = \frac{Y}{XZ}$. **5**

C) Compute the divided difference table for $f(x) = 3 \times 2^x$:

x	:	1.0	0.0	1.0	2.0	3.0
f(x)	:	1.5	3.0	6.0	12.0	24.0

Write down the Newton's polynomial $P_4(x)$. **5**

4. A) Assume that $f \in C^{N+1}[a, b]$ and $x_0, x_1, \ldots x_N \in [a, b]$ are N + 1 nodes. If $x \in [a, b]$ then prove that :

$f(x) = P_N(x) + E_N(x)$

Where $P_N(x)$ is a polynomial that can be used to approximate f(x) and $E_N(x)$ is the corresponding error in the approximation. **6**

B) Consider the system :

$5x - y + z = 10$

$2x + 8y - z = 11$

$-x + y + 4z = 3 \qquad P_0 = 0$

And use Gauss-Seidel iteration to find P_1, P_2, P_3. Will this iteration convergence to the solution ? **5**

C) Find the triangular factorization A = LU for the matrix :

$$A = \begin{bmatrix} 1 & 1 & 0 & 4 \\ 2 & -1 & 5 & 0 \\ 5 & 2 & 1 & 2 \\ -3 & 0 & 2 & 0 \end{bmatrix}.$$ **5**

5. A) Assume that $f \in C^5$ [a, b] and that x – 2h, x – h, x, x + h, x + 2h $\in$ [a, b]. Prove that :

$$f'(x) = \frac{-f(x)+2h+8f(x+h)-8f(x-h)+f(x-2h)}{12h}.$$ **6**

B) Let f(x) = x^3 find approximation for $f'(2)$. Use formula in Part (a) with h = 0.05. **5**

C) Use Newton's Method with the starting value (p_0, q_0) = (2.00, .25). Compute (p_1, q_1) (p_1, q_1), (p_2, q_2) for the nonlinear system;

$x^2 - 2x - y + 0.5 = 0$, $x^2 + 4y^2 - 4 = 0$. **5**

6. A) Assume that $X_j = X_0 + h_j$ are equally spaced nodes and $f_j = f(x_j)$. Derive the Quadrature formula $\int_{x_0}^{x_2} f(x) ; \frac{h}{3}(f_0 + 4f_1 + f_2)$. **6**

B) Let $f(x) = \frac{3x}{2^x}$. Use cubic Langranges interpolation based on nodes :

$x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$ to approximate f (1.5). **5**

C) Consider $f(x) = 2 + \sin\left(2\sqrt{x}\right)$. Investigate the error when the composite trapezoidal rule is used over [1, 6] and the number of subinterval is 10. **5**

7. A) Use Euler's method to solve the I.V.P :

   $y' = -ty$ over [0, 0.2] with $y(0) = 1$. Compute y_1, y_2 with $h = 0.1$. Compare the exact solution $y(0.2)$ with approximation. **8**

   B) Use the Runge-Kutta method of order N = 4 to solve the I.V.P. $y' = t^2 - y$ over [0, 0.2] with $y(0) = 1$, (take $h = 0.1$). Compare with $y(t) = -e^{-t} + t^2 - 2t + 2$. **8**

8. A) Use power method to find the dominant Eigen value and eigen vector for the matrix :

$$A = \begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$$ **8**

   B) Use Householder's method to reduce the following symmetric matrix to trigonal form : **8**

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

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M.A./M.Sc. (Semester – IV) Examination, 2011
MATHEMATICS
MT – 803 : Differential Manifolds
(New Course)
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. *: 1) Attempt* ***any five*** *questions.*
2) ***All*** *questions carry* ***equal*** *marks.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Let M be a k-manifold in $\mathbb{R}^n$ of class C^r. Let $\alpha_0 : U_0 \to V_0$ and $\alpha_1 : U_1 \to V_1$ be co-ordinate patches on M, with $W = V_0 \wedge V_1$ nonempty. Let $W_i = \alpha_i^{-1}(W)$. Then prove that $\alpha_1^{-1} \circ \alpha_0 : W_0 \to W_1$ is of class C^r and its derivative is non-singular. **8**

   b) Show that the unit circle is a 1-manifold in $\mathbb{R}^2$. **4**

   c) Find centroid of the parametrized curve $\alpha(t) = (a\cos t, a\sin t)$, $0 < t < \pi$. **4**

2. a) Let U be an open set in $\mathbb{R}^n$ and $f : V \to \mathbb{R}$ be of class C^r. Let $N =: \{x \in \mathbb{R}^n : f(x) \geq 0\}$. Show that N is an n-manifold in $\mathbb{R}^n$. **6**

   b) If ω and η are 1-forms in $\mathbb{R}^3$ given by $\omega = x_1x_2\,dx_1 + 3dx_2 - x_2x_3dx_3$ and $\eta = x_1\,dx_1 - x_2\,x_3^2\,dx_2 + 2x_1\,dx_3$. Find $d(\omega \wedge \eta)$. **6**

   c) Find the area of the 2-sphere $S^2(a)$ of radius a in $\mathbb{R}^3$. **4**

3. a) For every form ω, show that $d(d\omega) = 0$. **6**

   b) Define boundary of a manifold. Find the boundary of $E_t^2(a) = S^2(a) \cap H^3$. **6**

   c) Let $\beta : H' \to \mathbb{R}^2$ be the map $\beta(x) = (x, x^2)$; let N be the image set of β. Show that N is a 1-manifold in $\mathbb{R}^2$. **4**

4. a) Let M be a k-manifold of class C^r $\mathbb{R}^n$ and $p \in M$. Define the tangent space to M at p and show that the definition is independent of the choice of co-ordinate patch at p. **6**

b) Let V be a vector space of dimension n. If $A^k(V)$ is the space of alternating k-tensor on V, find a basis and dimension of $A^k(V)$. **6**

c) If $X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ a & b \end{pmatrix}$, find V(X). **4**

5. a) Let k > 1. If M is an orientable k-manifold with non-empty boundary, then prove that ∂M is orientable. **8**

b) Define exact differential form. If $w = \dfrac{x\,dx + y\,dy}{x^2 + y^2}$ is a 1-form on $\mathbb{R}^2 - 0$; then is w an exact form ? Justify. **8**

6. a) State and prove Green's theorem. **8**

b) If $T : V \to W$ is a linear transformation, and if f, g are alternating tensors on W, then with usual notation, prove that $T^*(f \wedge g) = T^* f \wedge T^* g$. **4**

c) Let F (X, Y) = $x_1y_3 + x_3y_1$. Express AF as a linear combination of elementary alternating tensors. **4**

7. a) Let A be an open set in $\mathbb{R}^k$ and $\alpha : A \to \mathbb{R}^n$ be of class C^∞. If w is an *l*-form defined in an open set of $\mathbb{R}^n$ containing $\alpha(A)$, then prove that $\alpha^*(dw) = d(\alpha^* w)$. **8**

b) Let A be the open unit ball in $\mathbb{R}^2$. Let $\alpha : A \to \mathbb{R}^3$ be given by the equation $\alpha(u, v) = (u, v, (1 - u^2 - v^2)^{1/2})$. If Y_α is the image set of α, then evaluate

$$\int_{Y_\alpha} \frac{1}{\|X\|^m}(x_1 dx_2 \wedge dx_3 - x_2 dx_1 \wedge dx_3 + x_3 dx_1 \wedge dx_2).$$ **8**

8. a) If G is symmetric, show that AG = 0. **4**

b) If ω and η are forms of orders k and *l* respectively, then prove that

$$d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^k \omega \wedge d\eta$$ **8**

c) State Stoke's theorem. **4**

M.A./M.Sc. (Semester – IV) Examination, 2011
MATHEMATICS
MT – 803 : Measure and Integration (Old)
(2005 Pattern)

Time : 3 Hours   Max. Marks : 80

*N.B.: i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Show that $\mu(E_1 \Delta E_2) = 0$ implies $\mu E_1 \mu E_2$ provided that E_1 and E_2 are in B. **4**

   b) If $E_i \in B$, then prove that $\mu\left(\bigcup_{1=1}^{\infty} E_1\right) \leq \sum_{1=1}^{\infty} \mu E_i$ **6**

   c) Let f be an extended real-valued function defined on X. Then show that the following statements are equivalent :

      i) $\{x : f(x) < \alpha\} \in B$ for each α

      ii) $\{x : f(x) \leq \alpha\} \in$ B for each α

      iii) $\{x : f(x) > \alpha\} \in$ B for each α

      iv) $\{x : f(x) \geq \alpha\} \in$ B for each α. **6**

2. a) State and prove Fatou's Lemma. **6**

   b) If f and g are non negative measurable functions and a and b non-negative constants then $\int af + bg = a\int f + b\int g$. We have $\int f \geq 0$ with equality only if f = 0 a.e. **6**

   c) Show that if $<E_i>$ is a sequence of measurable sets, $\mu\left(\bigcup_{1=1}^{\infty} E_i\right) < \infty$ and $\lim E_i$ exists, then show that $\mu(\lim E_i) = \lim \mu(E_i)$. **4**

3. a) State and prove Hahn Decomposition theorem. **6**

   b) Let E be a measurable set such that $0 < VE < \infty$. Then prove that there is a positive set A contained in E with $V(A) > 0$. **6**

   c) Let f be a continuous function and g a measurable function show that the composite function $(f_o g)$ is measurable. **4**

4. a) Let (X, B, μ) be a finite measure space and g an integrable functions such that for some constant M, $\left|\int g\phi d\mu\right| \leq M\|\phi\|_p$ for all simple functions ϕ. Then show that $g \in L^q$. **8**

   b) The class B of μ^*-measurable sets is a σ-algebra. If $\bar{\mu}$ is μ^* restricted to B, then show that $\bar{\mu}$ is a complete measure on B. **8**

5. a) Define product measure. Let $\{(A_i \times B_i)\}$ be a countable disjoint collection of measurable rectangles where union is a measurable rectangle A + B. Then prove that $\lambda(A + B) = \sum \lambda(A_i + B_i)$. **6**

   b) Define inner measure. Let B be a μ^*-measurable set with $\mu^* B > \infty$. Then show that $\mu_* B = \mu^* B$. **6**

   c) Show that every countable set has Hausdorff dimension zero. **4**

6. a) Let μ be a Bair measure on a locally compact space X and E a σ-bounded Baire set in X. Then prove that for $\in > 0$

   i) There is a σ-compact open set O with $E \subset O$ and $\mu(O \sim E) < \in$.

   ii) $\mu E = \sup\{\mu K : K \subset E, \text{K a compact } G_8\}$. **8**

   b) Let μ^* be a topologically regular outer measure on X. Then show that each Borel set is μ^*-measurable. **4**

   c) Define the following terms : **4**

   i) Outer measure

   ii) Baire measure

   iii) Hausdorff measure

   iv) σ-Algebra.

M.A./M.Sc. (Semester – I) Examination, 2011
MATHEMATICS
MT – 501 : Real Analysis – I
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) State and prove Fatou's lemma. **8**

b) Find the Fourier series for the function **6**

$$f(x) = 1 \text{ if } -\pi \le x < 0$$
$$= 0 \text{ if } 0 \le x \le \pi$$

c) State Ascoli-Arzela theorem. **2**

2. a) If f is a measurable function then show that $|f|$ is measurable. **3**

b) State and prove Lebesgue's monotone convergence theorem. **8**

c) Find the Bernstein polynomials of degree 1, 2 and 3 for f(x) = x. **5**

3. a) If $\{f_k\}_{k=1}^{\infty}$ is an orthonormal sequence in an inner product space V, then, show that for every $f \in V$, the series $\sum_{k=1}^{\infty} |< f, f_k >|^2$ converges and

$$\sum_{k=1}^{\infty} |< f, f_k >|^2 \le \|f\|^2 .$$ **8**

b) Show that Lebesgue measure of a cantor set is zero. **4**

c) Let E ≤ [a, b]. Show that E is measurable if and only if its characteristic function is measurable. **4**

4. a) Show that C ([a, b]) with supremum norm is complete. **6**

b) True or false ? Justify. There is no function defined on (0, 1) that is continuous at each rational point of (0, 1) and discontinuous at each irrational point of (0, 1). **5**

c) State and prove Hölder's inequality. **5**

5. a) Show that the closed unit ball of C([0, 1]) is not compact. **6**

   b) Let X be a complete metric space and T a contraction from X to X. Show that there exists a unique point $x \in X$ with $T_x = x$. **6**

   c) If $\{f_k\}_{k=1}^{\infty}$ is a sequence of measurable functions then show that sup f_k is measurable. **4**

6. a) If f is Riemann integrable then show that it is Lebesgue integrable and the values of two integrals are equal. **8**

   b) Show that the trigonometric system $\frac{1}{\sqrt{2\pi}}, \frac{\cos nx}{\sqrt{\pi}}, \frac{\sin mx}{\sqrt{\pi}}$ n, m = 1, 2,... is orthogonal in $L^2([-\pi, \pi], m)$. **6**

   c) State Stone-Weierstrass theorem. **2**

7. a) State and prove Cauchy-Schwarz inequality. **6**

   b) Show that the step functions are dense in $L^P(\mu)$ for P, $1 \leq P < \infty$. **5**

   c) Give an example of a sequence of functions which is pointwise convergent but not uniformly. Justify. **5**

8. a) If a subset of a metric space is compact then show that it is sequentially compact. **6**

   b) If f and g are measurable functions then show that f + g is measurable. **5**

   c) True or false ? Justify. If f is Riemann integrable and f = g almost everywhere, then g is also Riemann integrable. **5**

M.A./M.Sc. (Semester – I) Examination, 2011
MATHEMATICS
MT : 503 : Linear Algebra
(2008 Pattern)

Time : 3 Hours | Max. Marks : 80

*N.B. : i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Prove that if W_1 and W_2 are subspaces of vector space V. Then $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$. **6**

b) Let V be finite dimensional vector space over K then prove that the following statements are equivalent for a subset B of V.

i) B is a basis

ii) B is a minimal generating set, that is no subset of B can generate V.

iii) B is a maximal linearly independent set. **6**

c) Let $W = \left\{ [x_1 x_2 x_3 x_4]^t \in \mathbb{R}^4 / 2x_1 + 3x_2 = 4x_3 + x_4 \right\}$ show that W is a subspace of $\mathbb{R}^4$. Find basis of W and extend it to form a basis of $\mathbb{R}^4$. **4**

2. a) Let $T : V \to V^1$ be a Linear transformation. Prove that T is injective if and only if Ker(T) = {0}. **6**

b) Let V be a vector space over K and Let W_1 and W_2 be subspaces of V then show that

$(W_1 + W_2) / W_2 \simeq W_1 / W_1 \cap W_2$. **5**

c) If S and T are idempotent linear operators on a vector space V then show that

i) I – T is idempotent. I is identity operator.

ii) S + T is idempotent if ST = TS = 0. **5**

3. a) Let V be a finite vector space over K and let X and Y be subspaces of V. Prove that :

i) $(X + Y)^\circ = X^\circ \cap Y^\circ$

ii) $(X \cap Y)^\circ = X^\circ + Y^\circ$

Where S° is the annihilator of S. **6**

b) Prove that a Linearly independent subset of a finite dimensional vector space can be extended to form a basis of the vector space. **5**

c) Find the eigenvalues and eigenvectors of the matrix **5**

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$$

4. a) State and prove the primary decomposition theorem for finite dimensional vector space with linear operator T on V. **8**

b) If A and B are similar matrices with entries from field K then prove that $M_A(X) = M_B(X)$ i.e. minimal polynomials are same. Is converse true ? Justify. **5**

c) Determine whether $T : R^3 \to R^3$,

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+1 \\ y \\ z \end{bmatrix}$$ linear transformation. **3**

5. a) Prove that two triangulable $n \times n$ matrices are similar if and only if they have the same Jordan canonical form. **6**

b) Find the Jordan form for the matrix **6**

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) State Cayley-Hamilton theorem and illustrate by an example. **4**

6. a) If V is an inner product space over the field $\mathbb{R}$ and if $u, v \in V$ then prove that

i) $\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2$

ii) $\|u+v\|^2 - \|u-v\|^2 = 4 < u, v >$. **6**

b) Use Gram-Schmidt Orthonormalization process to obtain an orthonormal basis spanned by the following vectors in the standard inner product space $\mathbb{R}^3$.
$\{u_1 = (1, 1, 1), u_2 = (-1, 1, 0), u_3 = (1, 2, 1)\}$. **6**

c) Prove that a Jordan chain consists of Linear independent vectors. **4**

7. a) Let V and W be finite dimensional inner product spaces over F and Let $T \in L(V, W)$. Show that

i) Ker $T^* = (\mathrm{im}T)^{\perp}$ and

im $T^* = (\mathrm{Ker}T)^{\perp}$

ii) V = Ker T $\oplus$ imT^* and
W = im T $\oplus$ Ker T^*. **6**

b) Prove that the eigen values of a unitary operator have absolute value 1. **5**

c) Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in R^{3\times3}$. Find a polar decomposition of A. **5**

8. a) Let V be a vector space over K and let f, g be Linear form on V. Prove that the mapping $\phi(x+y) = f(x)g(y)$ is a bilinear form on V. **8**

b) State and prove Schur's theorem. **6**

c) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. **2**

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M.A./M.Sc. (Semester – I) Examination, 2011
MATHEMATICS
MT-505 : Ordinary Differential Equations
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B. : i) Attempt **any five** questions.*

*ii) Figures to the **right** indicate **full** marks.*

1. a) Let y_1 (x) and y_2 (x) be two solutions of the equation $y'' + p(x)y' + q(x)y = 0$ on the interval [a, b]. Prove that y_1 (x) and y_2 (x) are linearly dependent on [a, b] if and only if their Wronskian is identically zero. **8**

   b) Show that $y = c_1e^{2x} + c_2e^{2x}$ is the general solution of $y'' - 4y' + 4y = 0$. **4**

   c) If $y_1 = x$ is one solution of $x^2y'' + xy' - y = 0$ then find y_2 and the general solution. **4**

2. a) Explain the method of solving the linear non-homogeneous differential equation $y'' + P(x)y' + Q(x)y = R(x)$ by using variation of parameter. **8**

   b) Find the general solution of $y'' - y' - 2y = 4x^2$. **4**

   c) Find the normal form of the equation :

   $x^2y'' + xy' + (x^2 - p^2)y = 0$ **4**

3. a) Using the method of Frobenius series solve the differential equation :

   $2x^2y'' + x(2x+1)y' - y = 0$ **8**

   b) Find the particular solution of the differential equation $y'' - 3y' - 4y = e^{-x}$ using method of undetermined coefficients. **8**

P.T.O.

4. a) Let y (x) and z (x) be nontrivial solution of $y'' + q(x)y = 0$ and $z'' + r(x)z = 0$, where q (x) and r (x) are positive functions such that q (x) > r (x). Prove that y (x) vanishes at least once between any two successive zeros of z (x). **8**

b) Show that the series

$$y = 1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} - \frac{x^6}{1.2.3.4.5.6} + ...$$

is solution of equation $y'' = -y$. **4**

c) Find the regular singular points of the differential equation :

$(1 - x^2)y'' + y' + xy = 0$. **4**

5. a) Prove that the function E (x, y) = $ax^2 + bxy + cy^2$ is of negative type if and only if a < 0 and $b^2 - 4ac < 0$. **5**

b) Find the Liapunov function E (x, y) so that the point (0, 0) is a stable critical point of the system : $\begin{cases} \frac{dx}{dt} = -2xy \\ \frac{dy}{dt} = x^2 - y^3 \end{cases}$. **3**

c) If the roots m_1 and m_2 are real, distinct and of the same sign, then prove that critical point (0, 0) is a node where (0, 0) is a critical point of the system.

$$\begin{cases} \frac{dx}{dt} = a_1x + b_1y \\ \frac{dy}{dt} = a_2x + b_2y \end{cases}.$$ **8**

6. a) Determine the nature and stability properties of the critical point for the following system

$$\frac{dx}{dt} = 2x - 2y + 10$$

$$\frac{dy}{dt} = 11x - 8y + 49$$ **5**

b) For the following system

i) Find the critical points.

ii) Find the differential equation of the paths.

iii) Solve this equation to find the paths and

iv) Sketch a few of the paths.

$$\begin{cases} \dfrac{dx}{dt} = -y \\ \dfrac{dy}{dt} = x \end{cases}.$$ **8**

c) Replace the differential equation $y'' = 1$ by an equivalent system of first order equation. **3**

7. a) Show that

$$\begin{cases} x(t) = 2e^{4t} \\ y(t) = 3e^{4t} \end{cases} \text{ and } \begin{cases} x(t) = e^{-t} \\ y(t) = -e^{-t} \end{cases}.$$

are solutions of the homogeneous system.

$$\begin{cases} \dfrac{dx}{dt} = x + 2y \\ \dfrac{dy}{dt} = 3x + 2y \end{cases}$$

and they are linearly independent, write the general solution of this system. **8**

b) Solve the following system

$$\frac{dx}{dt} = x + y,\ \frac{dy}{dt} = 4x - 2y$$ **8**

8. a) Solve the initial value problem $\frac{dy}{dx} = x + y;\ \ y(0) = 1$. Using Picards method and compare the result with exact solution. **8**

b) The differential equation $y'' - 2xy' + 2Py = 0$ has the series solution of the form $y = \sum_{n=0}^{\infty} a_n x^n$. Show that the coefficient a_n are related by recursion formula

$$a_{n+2} = \frac{2(n-p)}{(n+2)(n+1)} a_n;\ n \geq 1.$$ **6**

c) Find the particular solution that satisfies the given initial condition;

$$\frac{dy}{dx} = 2\sin x \cos x.$$ **2**

M.A./M.Sc. (Semester – II) Examination, 2011
MATHEMATICS
MT-602 : Differential Geometry
(2008 Pattern)

Time: 3 Hours Max. Marks : 80

N.B. : *1) Attempt **any five** questions.*
*2) Figures at **right** indicate **full** marks.*

1. a) Let $S = f^{-1}(C)$ be n-surface in $\mathbb{R}^{n+1}$, where $f : U \to \mathbb{R}$ is such that $\nabla f(q) \neq 0$ for all $q \in s$ and let $\overline{X}$ be a smooth vector field on U whose restriction to S is a vector tangent field on S. If $\alpha : I \to U$ is any integral curve of X such that $\alpha(to) \in S$ for some $to \in I$ then prove that $\alpha(t) \in S$ for all $t \in I$. **6**

b) Find the integral curve through P = (1, 1) of the vector field $f(x_1, x_2) = (x_2, x_1)$. **5**

c) Sketch the level sets and graph of functions $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x_1, x_2) = x_2^2 - x_1^2$ at height C = –1, 0, 1. **5**

2. a) Let $\overline{X}$ be smooth vector field on an open set $U \subset \mathbb{R}^{n+1}$ and let $P \in U$. Then show that there exists an open interval I containing 0 and an integral curve $\alpha : I \to U$ of $\overline{X}$ such that

1) $\alpha(0) = P$

2) If $\beta : \bar{I} \to U$ is any other integral curve of $\overline{X}$ with $\beta(0) = P$, then $\bar{I} \subset I$ and $\beta(t) = \alpha(t)$ for all $t \in \bar{I}$. **8**

b) Show that graph of a function $f : U \to \mathbb{R}$, $U \subseteq \mathbb{R}^n$ is n-surface. **5**

c) Define : **3**

i) Tangent space

ii) Unit n-sphere

iii) Cylinder.

3. a) Sketch the surface of revolution obtained by rotating $\mathbb{C}$ about x_1 – axis where C is defined by $x_1^2 + (x_2 - 2)^2 = 1$. **6**

b) Let $S \subset \mathbb{R}^{n+1}$ be connected n-surface in $\mathbb{R}^{n+1}$. Show that there exist on S exactly two unit normal vector fields $\overline{N}_1$ and $\overline{N}_2$ and $\overline{N}_2(P) = -\overline{N}_1(P)$ for all $P \in S$. **5**

c) Find the length of parametrized curve $\alpha : [0, 2\pi] \to \mathbb{R}^3$ defined by $\alpha(t) = (\sqrt{2}\cos 2t, \sin 2t, \sin 2t)$. **5**

4. a) Let S be compact oriented n-surface in $\mathbb{R}^{n+1}$ exhibited as level set $f^{-1}(C)$ of the smooth function $f : \mathbb{R}^{n+1} \to \mathbb{R}$ with $\nabla f(p) \neq 0 \quad \forall P \in S$. Then show that Guass maps S onto the unit sphere S^n. **10**

b) Find velocity, acceleration and speed of parametrized curve.

$\alpha : I \to \mathbb{R}^4$ defined by $\alpha(t) = (\cos t, \sin t, 2\cos t, 2\sin t)$. **6**

5. a) For each a, b, c, d,$\in \mathbb{R}$ prove that parametrized curve

$\alpha(t) = (\cos(at + b), \sin(at + b), (t + d)$ is geodisic in the cylinder

$x_1^2 + x_2^2 = 1$ in $\mathbb{R}^3$ sketch figure for a = 0, c = 0. **6**

b) Let S be an n-surface in $\mathbb{R}^{n+1}$. Let $\alpha : I \to S$ be parametrized curve in S, let to $\in$ I and $\overline{v} \in S_{\alpha(to)}$. Then show that there exists a unique vector field V, tangent to S along α, which is parrallel and has $\overline{V}(to) = \overline{v}$. **8**

c) Define :

a) Guass-Kronecker curvature

b) Mean-Curvature. **2**

6. a) Show that Weingarton map is self adjoint. **6**

b) Compute Weingarton map for the sphere $x_1^2 + x_2^2 + x_3^2 + x_4^2 = r^2$ oriented by inward unit normal vector field (r > 0). **6**

c) Compute $\nabla_v f$ where $f : \mathbb{R}^3 \to \mathbb{R}$ is given by $f(x_1, x_2, x_3) = x_1\, x_2\, x_3$,

$\bar{v} = (1, 1, 1, a, b, c)$. **4**

7. a) Find curvature K of oriented plane curve defined by $x_1^2 - x_2^2 = 1$, $x_1 > 0$. **5**

b) Define 1-form. If η is 1 form on $\mathbb{R}^2 - \{0\}$ defined by

$\eta = \dfrac{-x_2}{x_1^2 + x_2^2} dx_1 + \dfrac{x_1}{x_1^2 + x_2^2} dx_2$ then show that line integral of η with respect to any closed peicewise smooth parametrized curve in $\mathbb{R}^2 - \{0\}$ is $2\pi K$ for some integer K. **8**

c) Sketch vector field on $\mathbb{R}^2 : \overline{X}(P) = (P, X(P))$ where X(P) = (0, 1). **3**

8. a) Let C be oriented plane curve. Then show that there exists a global parametrization of C iff C is connected. **12**

b) Let V be finite dimensional vector space with dot product and $L : V \to V$ be a self adjoint linear transformation on V, then show that there exists an orthonormal basis of V consisting of eigenvectors of L. **4**

M.A./M.Sc. (Semester – II) Examination, 2011
MATHEMATICS
MT – 604 : Complex Analysis
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B. : 1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) If $\sum a_n z^n$ is a given power series with radius of convergence R, then prove that

$$R = \lim \left|\frac{an}{a_{n+1}}\right| \text{ if this limit exists.}$$ **6**

b) Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ have the radius of convergence R > 0. Prove that the $\sum_{n=1}^{\infty} na_n z^{n-1}$ has radius of convergence R. **5**

c) Let $f : G \to \mathbb{C}$ be analytic and that G be connected. Prove that

1) If f (z) is real for all z in G then f is constant.

2) If $\bar{f}$ is analytic on G then f is constant. **5**

2. a) Let G and Ω be open subsets of $\mathbb{C}$. Let $f : G \to \mathbb{C}$ and $g : \Omega \to \mathbb{C}$ are continuous function such that $f(G) \subset \Omega$ and $g(f(z)) = z \;\; \forall\, Z \in G$. If g is differentiable and $g'(z) \neq 0$ then prove that f is differentiable and $f'(z) = \dfrac{1}{g'(f(z))}$. **8**

b) Define Möbius transformation. Prove that every Möbius transformation maps circles of $\mathbb{C}_\infty$ onto circles of $\mathbb{C}_\infty$. **5**

c) Evaluate the cross ratio (7+i, 1, 0, ∞). **3**

P.T.O.

3. a) State and prove Cauchy's Residue theorem. Hence evaluate $\int_C \frac{5z-2}{z(z-1)} dz$ where C is the circle $|z| = 2$ described in anticlockwise direction. **8**

b) Show that a Möbius transformation is a composition of translation, dilation and inversion. **4**

c) Evaluate $\int_r \frac{1}{z} dz \quad z \neq 0$ where $r(t) = e^{it} \quad 0 \le t \le 2\pi$. **4**

4. a) Let f be analytic in B (a ; R) then prove that $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ for $|z-a| < R$, $a_n = \frac{f^{(n)}(a)}{n!}$ and this series has radius of convergence $\geq$ R. **6**

b) Let f be analytic in the disk B (a ; R) and suppose that γ is a closed rectifiable curve in B (a ; R). Prove that $\int_r f = 0$. **5**

c) Prove that $\int_0^{2M} \frac{e^{is}}{e^{is} - z} ds = 2M$ if $|z| < 1$. **5**

5. a) Let γ be a closed rectifiable curve in $\mathbb{C}$. Prove that $\eta(\gamma ; a)$ is constant for 'a' belonging to a component of $G = \mathbb{C} - \{\gamma\}$. Also, prove that $\eta(\gamma ; a) = 0$ for a belonging to unbounded component of G. **6**

b) State and prove Liouville's theorem. **5**

c) Let G be a region and let f and g be analytic functions on G such that $f(z)\, g(z) = 0 \ \forall Z \in G$. Show that either $f \equiv 0$ or $g \equiv 0$. **5**

6. a) Let G be an open subset of the plane $f : G \to \mathbb{C}$ an analytic function. If γ is a closed rectifiable curve in G s.t $\eta(\gamma ; w) = 0$ for all w in $\mathbb{C} - G$ then prove that for a in G – {r}

$$\eta(\gamma ; a) f(a) = \frac{1}{2\Pi i} \int \frac{f(z)}{z-a} dz.$$ **8**

b) If G is simply connected and $f : G \to \mathbb{C}$ is analytic in G then prove that f has a primitive in G. **6**

c) State Goursat's Theorem. **2**

7. a) If G is a region with 'a' in G and if f is analytic on $G - \{a\}$ with a pole at z = a then prove that there is analytic function $g : G \to \mathbb{C}$ and a positive integer M such that $f(z) = \dfrac{g(z)}{(z-a)^m}$. **6**

b) State and prove Casorati-Weierstrass Theorem. **6**

c) Let $f(z) = \dfrac{z}{z^2 - 1}$ give the Laurent expansion of f (z) in the annulus ann (2 ; 1, 3). **4**

8. a) Let G be a region in $\mathbb{C}$ and f an analytic function on G. Suppose there is a constant. M such that $\lim\limits_{z \to a} \sup |f(z)| \leq M$ for all a in $\partial_\infty G$. Prove that $|f(z)| \leq M$ for all Z in G. **5**

b) State and prove Schwarz's Lemma. **6**

c) Show $\int\limits_{-\infty}^{\infty} \dfrac{x^2}{1+x^4}\, dx = \dfrac{M}{\sqrt{2}}$. **5**

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M.A/M.Sc. Examination, 2011
MATHEMATICS
MT – 605 : Partial Differential Equations
(2008 Pattern)

Time : 3 Hours | Max. Marks : 80

*N.B. : i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Find the general solution of $x^2p + y^2q = (x + y) z$. **4**

b) Solve : $\frac{dx}{y} = \frac{dx}{-x} = \frac{dz}{2x - 2y}$. **4**

c) Explain the method of solving the following first order partial differential equations :

a) $f(p, q) = 0$,

b) $f(z, p, q) = 0$. **8**

2. a) Reduce the equation $\frac{\partial^2 u}{\partial x^2} = x^2 \frac{\partial^2 u}{\partial y^2}$ to canonical form. **6**

b) Show that the pfaffian differential equation $y\,dx + x\,dy + 2z\,dz = 0$ is integrable and find its integral. **5**

c) Define compatible systems of first order partial differential equations. Give an example of compatible system of first order partial differential equations. **5**

3. a) Obtain D' Alembert's solution for the initial value problem :

$C^2 u_{xx} - u_{tt} = 0$

$u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$. **8**

b) Explain Jacobi's method to find complete integral of $f(x, y, z, u_x, u_y, u_z) = 0$. **8**

P.T.O.

4. a) Find the integral surface of the differential equation

   $(x-y)\,p+(y-x-z)\,q=z$ passing through the circle $z = 1$, $x^2 + y^2 = 1$. **8**

   b) Find the characteristic strips of the equation $xp + yq - pq = 0$ and obtain the equation of the integral surface through the curve $z=\frac{x}{2}$, $y=0$. **5**

   c) Classify second order linear partial differential equations. **3**

5. a) Find complete integral of $\left(p^2+q^2\right)y=qz$ by Charpits method. **8**

   b) Using variable separable method solve : $u_{xx}+u_{yy}=0,\quad 0<x<a,\ 0<y<b$

   with boundary conditions

   $u(x,0)=f(x)\quad 0\le x\le a$

   $u(x,b)=0\quad 0\le x\le a$

   $u(0,y)=0\quad 0\le y\le b$

   $u(a,y)=0\quad 0\le y\le b$. **8**

6. a) If u (x, y) is harmonic in a bounded domain D and continuous in $\overline{D}=D\cup B$.

   Prove that u attains its maximum on the boundary B of D. **8**

   b) Show that the solution of the Dirichlet problem if it exists then it is unique. **4**

   c) Show that the equation $u_{xx}-2x^2u_{xz}+u_{yy}+u_{zz}=0$ is Hyperbolic if $|x|>1$, Parabolic if $|x|=1$ and elliptic if $|x|<1$. **4**

7. a) State and prove Harnack's theorem. **8**

b) Prove that the solution u (x, t) of the differential equation $u_t - k\,u_{xx} = F(x, t)$

$0 < x < l$, $t > 0$ satisfying the initial condition $u(x, 0) = f(x)$ $0 \le x \le l$ and the boundary condition $u(0, t) = u(l, t) = 0$, $t \ge 0$ is unique. **5**

c) Eliminate the parameters α and β from the equation $z = \alpha x + \beta y$ and find corresponding partial differential equation. **3**

8. a) Solve $u_t - k\,u_{xx} = F(x, t)$, $-\infty < x < \infty$ $t > 0$ with initial conditions

$u(x, 0) = 0, \quad u_t(x, 0) = 0, \quad -\infty < x < \infty.$ **8**

b) State the condition that the one parameter family of surface f (x, y, z) = c is said to be equipotential. **3**

c) Show that the family of surfaces $f(x, y, z) = x^2 + y^2 + z^2 = c$, $c > 0$ is equipotential and find potential function. **5**

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[3921] – 301

M.A./M.Sc. (Semester – III) Examination, 2011
MATHEMATICS
MT-701 : Functional Analysis (New)
(2008 Pattern)

Time : 3 Hours   Max. Marks : 80

***Instructions**: i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Let N be a normed linear space. If $S = \{x \in N \mid \|x\| = 1\}$ is complete then, prove that N is a Banach space. **6**

   b) Define the norm and prove that norm is a continuous function. **4**

   c) Let N and N' be normed linear spaces. If N' is a Banach space then, prove that $B(N, N')$ is also a Banach space. **6**

2. a) If N is a normed linear space and x_0 is a non-zero vector in N, then prove that there exists a functional f_0 in N* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$. **6**

   b) Give an example of reflexive Banach space. **2**

   c) State and prove open mapping theorem. **8**

3. a) Prove that a non-empty subset X of a normed linear space N is bounded if and only if f(X) is a bounded set of numbers for each f in N*. **6**

   b) Let T be an operator on a Banach space B. Show that T has an inverse T^{-1} if and only if T* has an inverse $(T^*)^{-1}$, and that in this case $(T^*)^{-1} = (T^{-1})^*$. **6**

   c) Let $T : l_2 \to l_2$ be defined by $T(x_1, x_2, ...) = (0, x_1, x_2, ...)$. Find T*. **4**

4. a) State Cauchy Schwarz inequality, and prove that the inner product in a Hilbert space is jointly continuous. **6**

   b) Show that the Parallelogram law is not true in $l_1^n \ (n > 1)$. **6**

   c) Let $X = \mathbb{R}^2$. Find $M^{\perp}$ if $M = \{x\}$ where $x = (x_1, y_1) \neq 0$. **4**

P.T.O.

5. a) If M is a closed linear subspace of a Hilbert space, then prove that
   $H = M \oplus M^{\perp}$. **6**

   b) Let $\{e_1, e_2, ..., e_n\}$ be a finite orthonormal set in a Hilbert space H. If x is any vector in H, then prove that :

   $$\sum_{i=1}^{n} \left|(x, e_i)\right|^2 \leq \|x\|^2;$$

   further

   $$x - \sum_{i=1}^{n} (x, e_i)e_i \perp e_j \text{ for each j.}$$ **6**

   c) Let H be a Hilbert space and show that H* is also a Hilbert space with respect to the inner product $(f_x, f_y) = (x, y)$. **4**

6. a) Prove that an operator T on a Hilbert space H is self-adjoint if and only if (Tx, x) is real for all $x \in H$. **6**

   b) Prove that an operator T on a Hilbert space H is normal if and only if $\|T^*x\| = \|Tx\|$ for every $x \in H$. **6**

   c) Prove that the adjoint operation $T \to T^*$ on $\mathcal{B}(H)$ has the following properties :

   i) $(\alpha T)^* = \overline{\alpha} T^*$;

   ii) $(T_1T_2)^* = T_2^* T_1^*$. **4**

7. a) Show that unitary operators on a Hilbert space H from a group. **5**

   b) If P is a projection on a Hilbert space H with range M and null space N, then prove that $M \perp N$ if and only if P is self-adjoint; and in this case, $N = M^{\perp}$. **6**

   c) Show that a projection P on a Hilbert space H satisfies

   $O \leq P \leq I$.

   Under what conditions (i) P = O, (ii) P = I? **5**

8\. a) Let T be a normal operator on a Hilbert space H with spectrum $\{\lambda_1, \lambda_2, ..., \lambda_m\}$ and use the spectral resolution of T to prove the following : **6**

i) T is positive if and only if $\lambda_i \geq 0$ for each i;

ii) T is unitary if and only if $|\lambda_i| = 1$ for each i.

b) Let the dimension n of a Hilbert space H be 2, let B = $\{e_1 \cdot e_2\}$ be a basis for H, and assume that the determinant of a 2×2 matrix $[\alpha_{ij}]$ is given by $\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}$. If T is an arbitrary operator on H whose matrix relative to B is $[\alpha_{ij}]$, show that $T^2 - (\alpha_{11} + \alpha_{22})\,T + (\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})\,I = 0.$ Give verbal statement of this result. **4**

c) Show that $\left\{\frac{e^{inx}}{\sqrt{2\pi}}\right\}$ is an orthonormal set in $L_2[0, 2\pi]$. **6**

M.A./M.Sc. (Semester – III) Examination, 2011
MATHEMATICS
MT-702 : Ring Theory (New)
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B. : i) Answer **any five** questions.*

*ii) Figures to the **right** indicate **full** marks.*

1. a) Define a Boolean ring. Prove that every Boolean ring is commutative. **5**

b) If R and S are rings and $\phi : R \to S$ is a ring homomorphism then prove that : **7**

i) Kernel of ϕ is an ideal of R.

ii) The range of ϕ is a subring of S.

iii) $\dfrac{R}{\ker\phi} \simeq \phi(R)$.

c) If R and S are two non-zero rings with identity 1_R and 1_S respectively and if $\phi : R \to S$ is a non-zero homomorphism of rings such that $\phi(1_R) \neq 1_S$ then prove that $\phi(1_R)$ is a zero divisor in S. **4**

2. a) Prove that the commutative ring R with 1 is a field if and only if its ideals are only 0 and R. **5**

b) If R is a finite commutative ring with identity 1 then prove that the ideal I in R is prime ideal in R if and only if I is a maximal ideal in R. **6**

c) If R is a commutative ring with $1 \neq 0$ and if a is a nilpotent element in R then show that 1 + a is unit in R. **5**

3. a) Prove that the ring of Gaussian integers Z [i] is a Euclidean domain with respect to the norm, $N(a + ib) = a^2 + b^2$ for any $a + bi \in Z[i]$. **6**

b) Show that the ideal (2, x) is not the principal ideal in the polynomial ring Z [x]. **5**

c) If R is a ring of all continuous functions defined on [0, 1] and if $I = \left\{f \in R / f\left(\frac{1}{2}\right) = f\left(\frac{1}{3}\right) = 0\right\}$ then show that I is an ideal in R but not a prime ideal in R. **5**

4. a) If R is a principal ideal domain then prove that every irreducible element in R is a prime. **5**

b) Prove or disprove

If a and b are associates in the ring R then they are associates in any subring S of R. **5**

c) i) Show that the quadratic integer ring $Z\left[\sqrt{-5}\right]$ is not a unique factorization domain. **4**

ii) List the proper factors of 9 in $Z\left[\sqrt{-5}\right]$. **2**

5. a) If R is a commutative ring with 1 then prove that polynomial ring in more than one variable over R is not a principal ideal domain. **6**

b) If f (x) is a polynomial in F (x) then prove that $\frac{F[x]}{\langle f(x)\rangle}$ is a field if and only if f (x) is irreducible. **5**

c) If R is UFD and F is the field of fractions of R. Prove that if P (x) is a monic polynomial that is irreducible in R [x] then P (x) is irreducible in F [x]. **5**

6. a) Define Dedekind Hasse-norm. Prove that if R is a PID then there exist a multiplicative Dedekind-Hasse norm on R. **7**

b) If R is quadratic integer ring then prove that **9**

i) The element α is a unit in R if and only if $N(\alpha) = \pm 1$.

ii) If $N(\alpha)$ is a prime (in Z .) then α is irreducible in R.

iii) If R = z [i], a ring of Gaussian integers then is it true that 2 is a prime z [i] ? Justify.

7. a) State and prove Eisenstein's criterion for irreducibility and hence show that the polynomial $x^4 + 10x + 5$ in z [x] is irreducible. **6**

b) Find all monic irreducible polynomials of degree ≤ 3 in F_2 [x]. **5**

c) Prove that the quotient ring $\frac{z[i]}{\langle 1+i \rangle}$ is a field of order 2. **5**

8. a) Prove or disprove : A subring or a quotient of a UFD is UFD. **6**

b) Show that the polynomial f (x) = x in $\frac{z}{6z}[x]$ is not an irreducible polynomial. **5**

c) Construct the field with 49 elements. **5**

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M.A./M.Sc. (Semester – III) Examination, 2011
MATHEMATICS (Optional)
MT-704 : Measure and Integration (New)
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B : *i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Define the following terms :

i) σ-algebra ii) Measurable space
iii) Finite measure iv) Complete measure **4**

b) If E_i's are measurable sets with $\mu E_1 < \infty$ and $E_i > E_{i+1}$ then prove that

$$\mu\left(\bigcap_{1=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu E_n$$ **6**

c) Let (X, B) be a measurable space. If μ and ν are measures defined on B, then show that the set function λ defined on B by $\lambda E = \mu E + \nu E$ is also a measurable. **6**

2. a) Let C be a constant and the functions f and g are measurable. Prove that the functions f + c, cf, f + g, f.g and $f \vee g$ are measurable. **6**

b) Let μ be a complete measure and f is a measurable function. If $f = g$ a.e. then show that g is measurable and verify the result if complete is omitted. **6**

c) Show that if μ is complete, then $E_1 \in B$ and $\mu(E_1 \,\Delta\, E_2) = 0$ imply $E_2 \in B$. **4**

3. a) Let (f_n) be a sequence of non-negative measurable functions which converges almost everywhere to a function f and $f_n \le f$ for all n then prove that

$\int f = \lim \int f_n$. **6**

b) Let f and g are integrable functions and E is a measurable set, then show that

i) $\int_E (c_1 f + c_2 g) = c_1 \int_E f + c_2 \int_E g$

ii) If $|h| \le |f|$ and h is measurable then h is integrable.

iii) If $f \ge g$ a.e. then $\int f \ge \int g$. **6**

c) Show that, if f is integrable, then the set $\{x : f(x) \ne 0\}$ is of σ-finite measure. **4**

4. a) Let (X ; B) be a measurable space (μ_n) a sequence of measures that converges setwise to a measure μ and (f_n) a sequence of non-negative measurable functions that converges to the function f. Then show that $\int f.d\mu \leq \underline{\lim} \int f_n \, d\mu_n$. **8**

   b) State Hahn decomposition theorem and give an example to show that the Hahn decomposition need not unique. **4**

   c) Define positive set and show that the union of a countable collection of sets is positive. **4**

5. a) State and prove Radon-Nikodym theorem. **8**

   b) Let F be a bounded linear functional on $L^P(\mu)$ with $1 < P < \infty$. Then prove that there is a unique element $g \in L^q(\mu)$ with $\|F\| = \|g\|_q$. **4**

   c) For $1 \leq P \leq \infty$ the spaces $L^P(\mu)$ are Banach spaces and if $f \in L^P(\mu)$, $g \in L^q(\mu)$ with $\frac{1}{p} + \frac{1}{q} = 1$ then show that $f.g \in L^1(\mu)$ and $\int |f.g| \, d\mu \leq \|f\|_p . \|g\|_q$. **4**

6. a) Define outer measure. Let $< E_i >$ is a sequence of disjoint measurable sets and $E = UE_i$. Then prove that for any set A
   $\mu^*(A \cap E) = \Sigma \mu^*(A \cap E_i)$. **6**

   b) Define a measure on an algebra. If $A \in G$, then show that A is measurable with respect to μ^*. **6**

   c) Let X be a set consisting of two points. Construct an outer measure on X which is not regular. **4**

7. a) If μ is a finite Bair measure on the real line, then prove that its cumulative distribution function F is monotone increasing bounded function which is continuous on the right. Moreover $\lim_{x \to -\infty} F(x) = 0$. **8**

   b) Let E be a set in $R_{\sigma\delta}$ with $\mu \times v(E) < \infty$.
   Then prove that the function g defined by g (x) = VEx is a measurable function of x and $\int g d\mu = \mu \times V(E)$. **8**

8. a) Define inner measure. Prove that

   i) $\mu_* E \leq \mu^* E$ ii) If $E \in a$ (a measure on algebra) then $\mu_* E = \mu E$. **6**

   b) If $< E_i >$ is any disjoint sequence of sets then show that $\sum_{1=1}^{\infty} \mu_* E_i \leq \mu_* \left(\bigcup_{1=1}^{\infty} E_i \right)$. **6**

   c) Define Caratheodors outer measure and Hausdorff measure. **4**

M.A./M.Sc. (Semester – III) Examination, 2011
MATHEMATICS (Optional)
MT-704 : Mathematical Methods – I (Old)
(2005 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B. : i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1\. a) Find whether the following series converges or diverges **4**

$$\sum_{n=2}^{\infty} \frac{3^n - n^3}{n^5 - 5n^2}$$

b) Find the interval of convergence of the power series **4**

$$\sum_{n=1}^{\infty} \frac{(-)^n x^n}{n(n+1)}$$

c) Explain comparison test, integral test, ratio test for convergence of series of positive terms. **8**

2\. a) By using series expansion, show that

i) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$ **6**

ii) $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$

b) Show that if f (x) has period P, then average value of f is the same over any interval of length P. **6**

c) i) Define Gamma function

ii) State the relation between Beta and Gamma function. **4**

3\. a) Define even function and odd function, also sketch the graph and give geometrical interpretation of the functions, f (x) = x^2 and g (x) = sin x. **8**

b) Prove that $\overline{|P+1} + P\overline{|P}$, with usual notations. **4**

c) Find the complex form of the fourier series of the function

$$f(X) = \begin{cases} 0, & -\pi < X < 0 \\ 1, & 0 < X < \pi \end{cases}$$ **4**

4. a) Express the following integrals as Beta function and evaluate it

i) $J = \int_0^1 \frac{X^4}{\sqrt{1-X^2}} dX$ ii) $I = \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}}$ **8**

b) Prove that, $\Gamma(P)\,\Gamma(1-P) = \frac{\pi}{\sin \pi P}$ **8**

5. a) i) Define error function

ii) Show that, erf (–X) = – erf (X)

iii) Show that, erf(∞) = 1 **(2+3+3)**

b) Evaluate $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$ from Rodrigues formula. **8**

6. a) Show that $\int_{-1}^{1} [P_m(x)]^2 dx = \frac{2}{2m+1}$, with usual notations. **8**

b) Prove that $\frac{d}{dx}\left[X^P J_P(X)\right] = X^P J_{P-1}(X)$ where, $J_P(x)$ Bessel function of 1^{st} kind and order P **8**

7. a) If L [F(t) = F(s) then show that L [F(at)] = $\frac{1}{a} F\left(\frac{s}{a}\right)$ **4**

b) Evaluate, $\int_0^{\infty} te^{-2t} \cos t \, dt$ by using Laplace transform. **4**

c) State and prove convolution theorem for Fourier transform. **8**

8. a) Solve differential equation by Laplace transform

$$y''(t) - 3y'(t) + 2y(t) = 4e^{2t}$$

$$y(0) = -3, \; y'(0) = 5$$ **4**

b) Find the Fourier transform of $F(x) = \begin{cases} 1, & |X| < a \\ 0, & |X| > a \end{cases}$ **4**

c) A semi-infinite bar (extending from X = 0 to X = ∞) with insulated sides, is initially at the uniform temperature u = 0°. At t = 0, the end at X = 0 is brought to u = 100° and held there. Find the temperature distribution in the bar as a function of x and t. **8**

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B/I/11/145

[3921] – 305

M.A./M.Sc. (Semester – III) Examination, 2011
MATHEMATICS (Optional)
MT-705 : Graph Theory (New)
(2008 Pattern)

Time: 3 Hours Max. Marks: 80

*N.B. : 1) Answer **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) Prove that a graph is bipartite if and only if it has no odd cycle. **8**

   b) How many simple graphs are there on a fixed set of six vertices ? Draw all nonisomorphic simple graphs on a set of four vertices. **4**

   c) Show that the Peterson graph has girth 5. **4**

2. a) Prove that a graph is Eulerian if and only if it has at most one nontrivial component and its vertices all have even degree. **8**

   b) Prove that in an even graph, every non-extendible trail is closed. **4**

   c) Use Havel-Hakimi theorem to determine whether the sequence (3,3,3,3,3,2,2,1) is graphic. Provide a construction or proof of impossibility. **4**

3. a) Prove that the non-negative integers $d_1,d_2,...,d_n$ are the vertax degrees of some graph if and only if Σd_i is even. **5**

   b) Let T be a tree with average degree ‘a’. Determine n(T) in terms of ‘a’. **3**

   c) Prove that if G is a bipartite graph, then the maximum size of a matching in G equals the minimum size of a vertex cover of G. **8**

4. a) Show that if G is self-complementary, then $|V(G)| = 4n$ or $|V(G)| = 4n+1$, where n is an integer. **3**

   b) Prove that for an n-vertax graph G (with $n \geq 1$), G is connected and has no cycles if and only if G has no loops and has for each $u,v \in V(G)$, exactly one u-v path. **5**

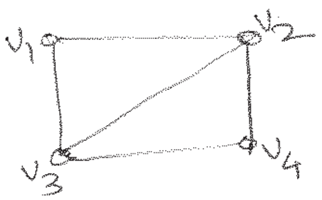
   c) Let $\tau(G)$ denote the number of spanning trees of a graph G. Prove that if $e \in E(G)$ is not a loop then $\tau(G) = \tau(G-e) + \tau(G.e)$. **8**

5. a) Prove that among trees with n vertices, the Wiener index $D(T) = \sum_{u,v} d(u, v)$ is minimized by stars and maximized by paths, both uniquely. **10**

b) Define Prüfer code. Let S= {1, 2, 3, 4, 5, 6, 7, 8} and f(T) = (744171). Use the Prüfer code algorithm to produce a tree with vertex set S. **6**

6. a) State the Matrix Tree theorem. **2**

b) Determine the number of spanning trees in the following graph by using the matrix tree theorem. **6**



c) Prove that if G is a connected graph, then an edge cut F is a bond if and only if G-F has exactly two components. **8**

7. a) Explain the Dijkstras algorithm for finding the shortest distance between any two vertices. **5**

b) Prove that in a connected weighted graph G, Kruskal's algorithm constructs a minimum weight spanning tree. **8**

c) Determine the perfect matchings in complete graph K_n. **3**

8. a) Prove that a matching M in a graph G is a maximum matching in G if and only if G has no M-augmenting path. **6**

b) Prove that in a graph G, $S \subseteq V(G)$ is an independent set if and only if if $\overline{S}$ is a vertex cover. **4**

c) Prove that if G is a 3-regular graph, then k(G) = K' (G). **6**

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M.A./M.Sc. (Semester – IV) Examination, 2011
MATHEMATICS
MT-801 : Field Theory (New Course) (2008 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B. : 1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) Let $f(x) = a_0 + a_1x + ... + a_{n-1} x^{n-1} + x^n \in \mathbb{Z} [x]$ be a polynomial. Prove that if $f(x)$ has a root α in $\mathbb{Q}$, then $\alpha \in \mathbb{Z}$ and $\alpha | a_0$. **4**

   b) Show that $f(x) = x^3 + 3x + 2$ in $\mathbb{Z}_7 [x]$ is irreducible over $\mathbb{Z}_7$. **2**

   c) Show that there exists an extension K of $\mathbb{Z}_3$ with nine elements, and exhibit its elements. **6**

   d) Find the minimal polynomial of $\left(\sqrt{2} - \sqrt[3]{2}\right)$. **4**

2. a) Let p(x) be a polynomial in F(x). Then, prove that there exist an extension E of F in which p(x) has a root. **6**

   b) Let K be any field. If K is algebraically closed proved that every irreducible polynomial in K(x) is of degree 1. **4**

   c) Prove that finite extension of prime degree is a simple extension. **4**

   d) Let E be a field extension of a field F such that [E : F] = 13, show that E is algebraic over F. **2**

3. a) State **true** or **false** with justification.

   i) Every algebraic field is finite extension. **5**

   ii) Let $E = F (u_1, u_2, ..., u_k)$ be a finitely generated extension of F where each u_i, i = 1 to k is algebraic over F. Then E is finite over F. **5**

   b) Find the splitting field E of a polynomial $f(x) = x^7 - 1$ over $\mathbb{Q}$ and also find $[E : \mathbb{Q}]$. **6**

4. a) If E is an extension of a field F with [E : F] = 2, then prove that E is normal extension of F. **3**

   b) If $f(x) \in F [x]$ is an irreducible polynomial over F then prove that all roots of f(x) have the same multiplicity. **5**

P.T.O.

c) Let E be a splitting field of a polynomial of degree n over a field F. Prove that $[E : F] \leq n!$

Further give an example in each of the following : **8**

i) $[E : F] = n\,!$ ii) $[E : F] < n$ and iii) $[E : F] = n$.

5\. a) Prove that any finite field F with P^n elements is the splitting field of $x^{P^n} - x$ in $F_p[x]$, where P is prime number. Hence or otherwise prove that any two finite fields with P^n are isomorphic. **5**

b) Prove that every finite separable extension is a simple extension. **7**

c) Give an example of a normal extension of a normal extension need not be normal. **4**

6\. a) Let E be a finite separable extension of a field F. Then prove that E is normal extension of F if and only if F is a fixed field of G(E/F). **8**

b) Let $E = \mathbb{Q}\left(\sqrt{2}, \sqrt{5}\right)$ be an extension of $\mathbb{Q}$. Find $G(E/\mathbb{Q})$. Find all subgroups of $G(E/\mathbb{Q})$ and their corresponding fixed fields. Draw Lattice diagram of subgroups of G (E/F) and fixed subfields of E. **8**

7\. a) Let E be a Galois extension of F and K be a subfield of E containing F. Then prove that K is normal extension of F : **8**

b) Let $E = \mathbb{Q}\left(\sqrt[4]{2}, i\right)$ and $F = \mathbb{Q}(i)$ then show that G(E/F) is cyclic and find its generators. **5**

c) Let K be a field of characteristic $p \neq 0$ and K be perfect, then prove that $K^P = K$. **3**

8\. a) Let E be a splitting field of $x^n - a$ in F(x), then prove that G(E/F) is solvable group. **8**

b) If $a > 0$ is constructible, then prove that $\sqrt{a}$ is constructible. **4**

c) Prove that it is impossible to construct a cube with a volume equal to twice the volume of the given cube using ruler and compass only. **4**

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M.A./M.Sc. (Semester – IV) Examination, 2011
MATHEMATICS
MT-801 : Algebraic Topology (Old)

Time : 3 Hours Max. Marks : 80

N.B. *: 1) Answer **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) Prove that the map $P : \mathbb{R} \to S^{\perp}$ given by $P(x) = (\cos 2\pi x, \ \sin 2\pi x)$ is a covering map. **6**

   b) Show that every contractible space is simply connected but converse is not true. **5**

   c) Let $r : X \to A$ be a retraction map of X onto A. If $a_{\cdot} \in A$, show that $r_* : \pi_1(X, a_0) \to \pi_1(A_, a_0)$ is subjective. **5**

2. a) Prove that the fundamental group of S^1 is isomorphic to the additive group in integers. **6**

   b) Let $P : S' \to S^{\perp}$ be given by $P(\tau) = \tau^n$. Prove that P is a covering map. **5**

   c) Show that if A is a retract of B^2, then every continuous map $f : A \to A$ has a fixed point. **5**

3. a) State and prove Brouwer's fixed-point theorem for disc. **6**

   b) Given a polynomial equation $x^n + a_{n-1}x^{n-1} + ... + a_1x + a_0 = 0$ with real coefficients. Show that if $|a_0| + |a_1| + ... + |a_{n-1}| < 1$, then all the roots of the equation lie interior to the unit ball B^2. **5**

   c) Show that if $g : S^2 \to S^2$ is continuous and $g(x) \neq g(-x)$ for all x, then g is surjective. **5**

4. a) Prove that given two bounded polygonal regions in $\mathbb{R}^2$, there exists a line in $\mathbb{R}^2$ that bisects each of them. **6**

   b) Prove that the inclusion map $j : S^n \to \mathbb{R}^{n+1} - 0$ induces an isomorphism of fundamental groups. **5**

c) Find the fundamental groups of **5**

i) $\left\{ x \in \mathbb{R}^2 / \|x\| > 1 \right\}$

ii) $S' \cup (\mathbb{R} \times 0)$

iii) $S^{\perp} \cup (\mathbb{R}_+ \times \mathbb{R})$.

5. a) State and prove the fundamental theorem of algebra using fundamental groups. **8**

b) Prove that for $n \geq 2$, the n-sphere S^n is simply connected. **8**

6. a) Prove that the fundamental group of figure eight is not abelian. **8**

b) Compute the fundamental groups of $S^1 \times B^2$ and $S^1 \times S^2$. **4**

c) i) Show that $\mathbb{R}^{\perp}$ and $\mathbb{R}^n$ are not homeomorphic if $n > \perp$.

ii) Show that $\mathbb{R}^2$ and $\mathbb{R}^n$ are not homeomorphic if $n > 2$. **4**

7. a) Let C be a simple closed curve in S^2. Prove that C separates S^2. **8**

b) Let X be the space obtained from a finite collection of polygonal regions by pasting edges together according to some labelling scheme. Prove that X is a compact Housdorff space. **8**

8. a) Let $P : E \to B$ and $P' : E' \to B$ be covering maps; let $P(e_0) = P'(e'_0) = b_0$. Prove that there is an equivalence $h : E \to E'$ such that $h(e_0) = e'_0$ if and only if the groups :

$H_0 = P_*(\pi_1(E, e_0))$ and $H'_0 = P'_*(\pi_1(E', e'_0))$ are equal. Further, prove that if h exists then it is unique. **8**

b) Let $P : E \to B$ be a covering map, with E simply connected. Prove that given any covering map $r : Y \to B$, there is a covering map $q : E \to Y$ such that roq = P. **4**

c) Give an example of a path connected and locally path connected space which has no covering space. **4**

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M.A./M.Sc. (Sem. IV) Examination, 2011
MATHEMATICS
(2008 Pattern)
MT - 802 : Combinatorics (New)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) How many integers between 1000 and 10,000 are there (leading zeros not allowed) with

   a) Repetition of digits allowed but with no 2 or 4 ?

   b) Distinct digits and at least one of 2 and 4 must appear ? **6**

   B) Prove by combinatorial argument that C(r, r) + C(r + 1, r) + C(r + 2, r) + ... + C(n, r) = C(n + 1, r + 1).

   Hence evaluate the sum $1^2 + 2^2 + ... + n^2$. **6**

   C) Find all derangements of 1, 2, 3, 4, 5 with the help of associated chessboard of darkened squares. **4**

2. A) What fraction of all arrangements of INSTRUCTOR have three consecutive vowels ? **6**

   B) How many numbers between 0 and 10,000 have a sum of digits equal to 13 ? **6**

   C) Among 40 toy robots, 28 have a broken wheel or are rusted but not both, 6 are not defective, and the number with a broken wheel equals the number with rust. Find how many robots are rusted. **4**

3. A) How many ways are there to select 300 chocolate candies from seven types if each type comes in boxes of 20 and if at least one but not more than five boxes of each type are chosen ? **6**

   B) Find ordinary generating function whose coefficient a_r equals $3r^2$. Hence, evaluate the sum $0 + 3 + 12 + ... + 3n^2$. **6**

   C) How many ways are there to distribute 15 identical objects into four different boxes if the number of objects in box 4 must be a multiple of 3 ? **4**

4. A) Use generating functions to find the number of ways to select 10 balls from a large pile of red, white and blue balls if the selection has at most two red balls. **6**

B) How many ways are there to form a committee of 10 mathematical scientists from a group of 15 mathematicians, 12 statisticians and 10 operations researchers with at least one person of each different profession on the committee ? **6**

C) Find generating function for modeling the number of 5-combinations of the letters M, A, T, H in which M and A can appear any number of times but T and H appear at most once. Which coefficient in this generating function do we want ? **4**

5. A) How many r digit quaternary sequences are there in which the total number of 0's and 1's is even ? **6**

B) Using inclusion-exclusion principle, find the number of ways to distribute 25 identical balls into 6 distinct boxes with at most 6 balls in any of the first three boxes. **6**

C) Solve the recurrence relation

$a_n = 2a_{\frac{n}{2}} + 2 , n \geq 4$ with $a_2 = 1$. **4**

6. A) Solve the recurrence relation

$a_n = 2\ a_{n-1} + 2^n$ with $a_0 = 1$,

using generating functions. **6**

B) How many numbers between 1 and 280 are relatively prime to 280 ? **6**

C) How many ways are there to distribute eight different toys among four children if the first child gets at least two toys ? **4**

7. A) Find a recurrence relation for the ways to distribute n identical balls into K distinct boxes with between two and four balls in each box. Repeat the problem with balls of three colors. **6**

B) Solve the recurrence relation

$a_n = a_{n-1} + 3n^2$ with $a_0 = 10$. **6**

C) How many arrangements of the letters in MATHEMATICS are there in which TH appear together but the TH is not immediately followed by an E ? **4**

8. A) Seven dwarfs D_1, D_2, D_3, D_4, D_5, D_6, D_7 each must be assigned to one of seven jobs in a mine, J_1, J_2, J_3, J_4, J_5, J_6, J_7. If D_1 cannot do jobs J_1 or J_3 ; D_2 cannot do J_1 or J_5 ; D_4 cannot do J_3 or J_6 ; D_5 cannot do J_2 or J_7 ; D_7 cannot do J_4; D_3 and D_6 can do all jobs. How many ways are there to assign the dwarfs to different jobs ? **8**

B) Find recurrence relation for the number of n-digit ternary sequences with an even number of 0's and an even number of 1's. **8**

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M.A./M.Sc. (Semester – IV) Examination, 2011
MATHEMATICS
(2005 Pattern) (Old Course)
MT 802 : Hydrodynamics

Time : 3 Hours Max. Marks : 80

Instructions : *1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) Explain Lagranges method of description and hence derive equation of continuity. **6**

b) A two dimensional incompressible flow field has the x-component of velocity given by $u = \frac{xy}{y - x}$. Determine y-component, v of the velocity. Is this flow irrotational ? Justify. **8**

c) Define stream function. **2**

2. a) A three dimensional field is given by $u = xy^2t$, $v = \frac{1}{3}y^3t^3$, $w = \frac{1}{2}xyz^2t^2$. Determine the acceleration at point (1, 2, 3) at t = 1 sec. **8**

b) The velocity components of a flow inside an elliptic cylinder $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are $u = -\frac{2ky}{b^2}, v = \frac{2kx}{a^2}$. Find the circulation about the cylinder. **8**

3. a) Given the velocity $\bar{q} = (1 + t)x\hat{i} + (2 + t)y\hat{j}$, find the equaiton of path line passing through (1, 2, 0) at t = 0. **6**

b) Show that the complex potential $w = \phi + i\psi$ where ϕ is velocity potential and ψ is stream function satisfies Cauchy Riemann conditions. **5**

c) Find the stream function ψ for an irrotational flow whose velocity components in cylindrical co-ordinates are

$$u(r,\theta) = U\left(1-\frac{a^2}{r^2}\right)\cos\theta,\ V = -U\left(1+\frac{a^2}{r^2}\right)\sin\theta.$$ **5**

4. a) The velocity components of a certain flow are given as u = a(x + y), v = b(x^2 – y^2) + 6y, w = –2dz where a, b, d are constants. Represent the motion as the sum of rotation and deformation of fluid element. **8**

b) State and prove Kutta-Jouskowski theorem. **8**

5. a) Determine complex potential of a two dimensional vortex motion. Define vortex pair and find the complex potential of vortex pair. **8**

b) Show that the kinetic energy of an infinite mass of liquid of density ρ moving irrotationally is given by $-\frac{1}{2}\rho \int_S \phi \frac{\partial \phi}{\partial n}\, ds$ where ϕ denotes single valued velocity potential. **8**

6. a) Show that stress tensor is symmetric. **8**

b) Discuss the flow due to a circular cylinder of mass m moving with velocity U. **8**

7. a) Explain volumetric deformation and simple shear. **6**

b) Compute the components of rate of deformation of a flow with velocity components u = 0, v = 0, $w = w(r,\theta)$. **10**

8. a) Obtain the relation between stress and rate of strain components. **10**

b) State Bernoulli's theorem for steady and unsteady flow. **4**

c) State circle theorem. **2**

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[3921] – 405

M.A./M.Sc. (Semester – IV) Examination, 2011
MATHEMATICS (2008 Pattern)
MT-805 : Lattice Theory (New)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Answer **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) Let the algebra $< L, \wedge, v >$ be a lattice. Set $a \leq b$ iff $a \wedge b = a$. Then show that $< L, \leq >$ is lattice. **6**

   b) Prove that I is a prime ideal of lattice L if and only if there is a homomorphism ϕ of L onto C_2 with $I = \phi^{1}(0)$. **5**

   c) Let L be a lattice and I be non-empty sub-set of L. Prove that I is an ideal, if and only if $a, b \in I$. Implies that $a \vee b \in I$ and $a \in I$, $x \in L$, $x \leq a$. Imply that $x \in I$. **5**

2. a) Prove that every Homomorphic image of a lattice L in Isomorphic to suitable quotient lattice of L. **6**

   b) Let I be ideal and D be a dual ideal, if $I \cap D \neq \phi$, then show that $I \cap D$ is a convex sub-lattice and every convex sub-lattice can be expressed in this form uniquely. **6**

   c) Prove that dual of Distributive lattice is a distributive. **4**

3. a) Let L and K be lattices, let θ and ϕ be a congruence relations on L and K respectively. Define the relation $\theta \times \phi$ of $L \times K$ by $(a, b) \equiv (c, d)\ (\theta \times \phi)$ if and only if $a \equiv c\ (\theta)$ and $b \equiv d\ (\phi)$. Then prove that $\theta \times \phi$ is a congrunce relation of $L \times K$ more over every congruence relation of $L \times K$ is of this form. **6**

   b) Let L be a pseudocomplemented meet-semi-lattice and let $a, b \in L$. Then verify the formula $(a \wedge b)^* = (a^{**} \wedge b)^* = (a^{**} \wedge b^{**})^*$. **5**

   c) Prove that if L is finite then, L and Id (L) [ideal lattice of L] are isomorphic. **5**

4. a) Prove that a Lattice is modular if and only if it does not contain a pentagon. **8**

   b) State and prove Nachbin theorem. **8**

5. a) State and prove Hashimoto theorem. **8**

   b) A lattice is distributive if and only if it is isomorphic to ring of sets. **8**

6. a) Show that every element of finite distributive lattice has a unique Irredundant representation as a join of Join-irreducible elements. **6**

   b) Let L be a distributive lattice and let $a \in L$. Show that the map $\phi: x \to \langle x \wedge a, x \vee a \rangle, x \in L$, is an embedding of L into (a] × [a). **5**

   c) Show that in a bounded distributive lattice, if an element has a complement then it also has a relative complement in any interval containing it. **5**

7. a) State and prove Stone's separation theorem for distributive lattice. **8**

   b) Prove that every ideal of distributive lattice is a standard ideal and conversely. **5**

   c) Show that $N5 \cong L \times K$ implies that the lattice L or K has only one element. **3**

8. a) Prove that in a Boolean lattice an ideal is maximal iff it is prime. **6**

   b) Prove that complemented elements of a lattice form a sublattice. **5**

   c) Prove that every distributive lattice is modular but not conversely. Find the smallest modular but non distributive lattice. **5**

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[3921] – 405

M.A./M.Sc. (Semester – IV) Examination, 2011
MATHEMATICS
MT-805 : Field Theory (Old)
(2005 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B.: 1) Answer **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) Let α be algebraic over a field k. Then prove that $k(\alpha) = k[\alpha]$ and $k(\alpha)$ is a finite extension of k.

   Further, show that the degree $[k(\alpha) : k]$ is equal to the degree of $I_n(\alpha, k, X)$. **7**

   b) Let $E = F(\alpha)$ where α is algebraic over field F, of odd degree. Show that $E = F(\alpha^2)$. **4**

   c) Show that $\sqrt{3} + i$ is algebraic over $\mathbb{Q}$, of degree 4. **5**

2. a) Let k be a field. Prove that there exists an algebraically closed field containing k as a subfield. **10**

   b) Let E, F be two finite extensions of a field k, contained in a larger field k. If [E: k] and [F : k] are relatively prime then show that [EF : k] = [E : k] [F:k]. **6**

3. a) Let E be a finite extension of a field k. Prove that there exists an element $\alpha \in E$ such that $E = k(\alpha)$ if and only if there exists only a finite number of fields F such that $k \subset F \subset E$. Further, show that if E is separable over k, then there exists an element $\alpha \in E$ such that $E = k(\alpha)$. **10**

   b) Let $E = \mathbb{Q}(\sqrt{2})$ and $F = \mathbb{Q}\left(2^{1/4}\right)$. Show that

   i) F is a normal extension of E.

   ii) E is a normal extension of $\mathbb{Q}$.

   iii) F is not a normal extension of $\mathbb{Q}$. **6**

4. a) Let p be a prime and $q = p^n$. Let $\mathbb{F}_q$ be the finite field with q elements. Consider $\mathbb{Q} : \mathbb{F}_q \to \mathbb{F}_q$ defined by $\mathbb{Q}(x) = x^p$. Show that $\mathbb{Q}$ is a one-one homomorphism. Further, show that $\mathbb{Q}$ is surjective. Also, show that the group of automorphisms of $\mathbb{F}_q$ is cyclic of degree n, generated by $\mathbb{Q}$. **8**

b) Find the splitting field of the polynomial $x^6 + x^3 + 1$ over $\mathbb{Q}$ and determine its degree over $\mathbb{Q}$. **5**

c) Let $E = \mathbb{Q}(\alpha)$, where α is a root of the equation $\alpha^4 - \alpha^3 + \alpha^2 - \alpha + 5 = 0$. Express $(\alpha - 1)^{-1}$ in the form $a\alpha^2 + b\alpha + c$, where $a, b, c \in \mathbb{Q}$. **3**

5. a) Let K be a field and let G be a finite group of automorphisms of K, of order n. Let $k + k^G$ be the fixed field. Prove that K is a finite Galois extension of k, and its Galois group is G. **10**

b) Determine the Galois group of the polynomial $f(x) = x^3 - x + 1$ over the rational numbers. **6**

6. a) Let τ be a primitive n-th root of unity. Prove that $\mathbb{Q}(\tau)/\mathbb{Q}$ is a Galois extension. Determine Galois group of $\mathbb{Q}(\tau)$ over $\mathbb{Q}$. **10**

b) Let τ be a primitive n-th root of unity. Let $K = \mathbb{Q}(\tau)$

i) If $n = p^r (r \geq 1)$ is a prime power, show that $N_{k/\mathbb{Q}}(1-\tau) = p$.

ii) If n is composite, divisible by at least two primes then show that $N_{k/\mathbb{Q}}(1-\tau) = 1$. **6**

7. a) Let G be a monoid and k a field. Let $x_1, ..., x_n$ be distinct characters of G in K. Prove that they are linearly independent over K. **8**

b) Let $E = \mathbb{Q}(2^{1/3})$. Find the dual basis of $\{1, 2^{1/3}, 2^{2/3}\}$. **4**

c) Define a cyclic extension. Give an extension of $\mathbb{Q}$ which is cyclic extension of degree 4. **4**

8. a) Let E be a separable extension of k. Prove that E is solvables by radicals if and only if E/k is solvable. **10**

b) Let τ be a primitive P^{th} root of unity. Prove that $\mathbb{Q}(\sqrt{P})$ is contained in $\mathbb{Q}(\tau, \sqrt{-1})$. **6**

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M.A./M.Sc. (Semester – IV) Examination, 2011
MATHEMATICS
MT – 806 : Lattice Theory (Old)
(2005 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B: 1) Answer **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) Let the poset $L = < L; \leq >$ be a lattice. Set $a \wedge b = \inf\{a,b\}$, $a \vee b = \sup\{a, b\}$. Then show that the algebra $L < L; \wedge, \vee >$ is a lattice. **5**

   b) Prove that I is a proper ideal of lattice L if and only if there is a join homomorphism ϕ of L on to C_2 with $I = \phi^{-1}(0) = \{x \in L \int \phi(x) = 0\}$ **6**

   c) Prove that every distributive lattice is modular but conversely. **5**

2. a) Prove that every Homomorphic image of a lattice L in Isomorphic to suitable quotient lattice of L. **5**

   b) If θ is a congruence relation of lattice L then prove that for any $a \in L$ $[a]\theta$ is a convex sub-lattice. **5**

   c) Let I be ideal and D be a dual ideal. If $I \cap D \neq \phi$, then show that $I \cap D$ is a convex sub-lattice and every convex sub-lattice can be expressed in this form uniquely. **6**

3. a) Prove that the identity $(x \wedge y) \vee (x \wedge z) = (x \wedge (y \vee (x \wedge z))$ is equivalent to the condition :

   $x \geq z$ implies that $(x \wedge y) \vee z = x \wedge (y \vee z)$. **4**

   b) Let L be a pseudocomplemented meet-semi-lattice. Prove that the lattice $S(L)=\{a^*/a \in L\}$ is a Boolean. **6**

   c) Prove that a lattice L is distributive iff the identity :

   $(x \wedge y) \vee (y \wedge z) \vee (z \wedge x) = (x \vee y) \wedge (y \vee z) \wedge (z \vee x)$ holds in L. **6**

4. a) Prove that a lattice is modular if and only if it does not contain a pentagon . **8**

   b) State and prove Nachbin theorem. **8**

5. a) State and prove Hashimoto theorem. **8**

   b) Prove that a finite lattice is distributive if and only if it is isomorphic to ring of sets. **8**

6. a) Show that every element of finite distributive lattice has a unique Irredundant representation as a join of join-irreducible elements. **6**

   b) Let L be a distributive lattice and let $a \in L$. Show that the map $\phi : x \rightarrow < x \wedge a, x \vee a >, x \in L$ is an embedding of L into $(a] \times [a)$. **5**

   c) Show that every ideal of distributive lattice is intersection of all prime ideals containing it. **5**

7. a) Let L is a bounded distributive lattice with $0 \neq 1$. Prove that L is a Boolean if P(L) is unordered. **6**

   b) Prove that every standard and dually standard element is neutral and every standard element is distributive. **6**

   c) Show that $N_5 \cong L \times K$ implies that the lattice L or K has only one element. **4**

8. a) Prove that every maximal chain of a finite distributive lattice L is of length IJ(L)I. **5**

   b) Prove that complemented element of a lattice forms a sub-lattice. **6**

   c) Prove that any modular lattice can be embedded in a complete modular lattice. **5**

M.A./M.Sc. (Semester – IV) Examination, 2011
MATHEMATICS
MT-807 : Combinatorics (Old)
(2005 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B.: 1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. A) a) How many even five-digit numbers (leading zeros not allowed) are there ? **6**

b) How many five-digit numbers are there with exactly one 3 ?

c) How many five-digit numbers are there that are the same when the order of their digits is inverted (e.g. 15251) ?

B) How many arrangements of INSTRUCTOR are there in which there are exactly two consonants between successive pairs of vowels ? **6**

C) How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 28$ with $x_i \geq i$ (i = 1, 2, 3, 4, 5) ? **4**

2. A) Prove by combinatorial argument that

$C(r, r) + C(r + 1, r) + C(r + 2, r) + ... + C(n, r) = C(n + 1, r + 1)$

Hence evaluate the sum

$1\times2\times3 + 2\times3\times4 + ... + (n - 2)(n - 1)n.$ **6**

B) How many ways are there to place an order for 12 chocolate sundaes if there are 5 types of sundaes and at most 4 sundaes of one type are allowed ? **6**

C) How many numbers greater than 30,00,000 can be formed by arrangements of 1, 2, 2, 4, 6, 6, 6 ? **4**

3. A) How many arrangements of MISSISSIPPI are there with no pair of consecutive S’s ? **6**

B) Solve the recurrence relation

$a_n = a_{n-1} + n(n-1)$ with $a_0 = 3$. **6**

C) How many r-digit ternary sequences are there with an even number of 0’s and even number of 1’s ? **4**

4. A) Find ordinary generating function whose coefficient a_r equals 3r + 7. Hence evaluate the sum 7 + 10 + 13 + ... + (3n + 7). **6**

B) How many n-digit numbers are there with at least one of the digits 1 or 2 or 3 absent ? **6**

C) Find a recurrence relation for the number of n-digit binary sequences with no pair of consecutive 1’s. **4**

5. A) Using Generating functions, solve the recurrence relation :

$a_n = a_{n-1} + 2$ with $a_0 = 1$. **6**

B) A school has 200 students with 80 students taking each of the three subjects : trigonometry, probability and basket-weaving. There are 30 students taking any given pair of these subjects, and 15 students taking all three subjects **6**

a) How many students are taking none of these three subjects ?

b) How many students are taking only probability ?

C) Give combinatorial proof of :

$$\binom{2n}{2} = 2\binom{n}{2} + n^2.$$ **4**

6. A) How many arrangements are there of a, a, a, b, b, b, c, c, c without three consecutive letters the same ? **6**

   B) Suppose a school with 120 students offers Yoga and Karate. If the number of students taking Yoga alone is twice the number taking Karate (possibly Karate and Yoga) and if 25 more students study neither skill than study both skills, and if 75 students take at least one skill, then how many students study Yoga ? **6**

   C) Find two different chessboards (not row or column rearrangements of one another) that have the same rook polynomial. Write the rook polynomial. **4**

7. A) State and prove Burnside's theorem. **8**

   B) A computer dating service wants to match four women each with one of five men. If **8**

   woman 1 is incompatible with men 3 and 5;

   women 2 is incompatible with men 1 and 2;

   women 3 is incompatible with man 4; and

   women 4 is incompatible with men 2 and 4,

   how many matches of the four women are there ?

8. A) State and prove the Inclusion-exclusion formula. **6**

   B) If a necklace can be made from beads of three colors : black, white and red, how many different necklaces with n beads are there ? **6**

   C) A baton is painted with equal sized cylindrical bands using black or white colors. If the baton is unoriented when spun in the air, how many different 2-colorings of the baton are possible if baton has 4 bands ? **4**

M.A./M.Sc. (Semester – III) Examination, 2011
MATHEMATICS (Optional)
MT-703 : Mechanics (New Course) (2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. *: 1) Answer **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. A) Define the following :

   i) Virtual displacement

   ii) Degrees of freedom

   iii) Scleronomous constraint

   iv) Cyclic coordinate. **8**

   B) Explain the following :

   i) D'Alembert's principle

   ii) Hamilton's principle. **4**

   C) If L is a Lagrangian for a system of n degrees of freedom satisfying Lagrange's equations of motion. Show that $L' = L + \frac{dF}{dt}(q_1, ..., q_n, t)$ also satisfies Lagrange's equations where F is any arbitrary but differentiable function of its arguments. **4**

2. A) Derive Lagrange's equations of motion from Hamilton's principle. **8**

   B) Let $\alpha(q, \dot{q}, t) = \frac{m}{2}\left(\dot{q}^2 \sin^2 wt + q\dot{q}\sin^2 wt + q^2 w^2\right)$, where m, w are constants. Find the Hamilton's equations of motion. If the Hamiltonian conserved ? **8**

3. A) Find the degrees of freedom of

i) rigid body

ii) conical pendulum. **4**

B) If Lagrangian $\alpha\left(x, \dot{x}\right) = e^{\gamma t}\left[\frac{1}{2} m\dot{x}^2 - \frac{1}{2} kx^2\right]$, γ, k are constants. Find Lagrange's equation of motion. **3**

C) Show that if the Hamiltonian is not an explicit function of time, then it is a constant of motion. **3**

D) Show that in a conservative field of motion the total energy is constant. **6**

4. A) State and prove Euler's theorem on rigid body motion. **6**

B) Show that the central force motion of two bodies about their centre of mass can always be reduced to an equivalent one-body problem. **5**

C) Prove the Kepler's second law of planetary motion which states that the radius vector sweeps out equal areas in equal times. **5**

5. A) Explain the fact that finite rotations do not commute. **4**

B) Show that if the law of central force is an inverse square law of attraction then the path of the particle is a conic. **6**

C) Explain Euler angles graphically. **6**

6. A) Define infinitesimal rotations and show that they can be represented vectorially. **6**

B) Prove invariance of Poisson brackets under canonical transformations. **6**

C) Is the following transformation canonical ?

$Q = \log\left(\frac{1}{q}\sin p\right), P = q\cot p$ **4**

7. A) Find canonical transformation generated by $F_3 = -(e^Q - 1)^2 \tan p$. **5**

B) Define Poisson brackets and show that any three dynamical variables u, v, w satisfy the Jacobi identity :

$[u, [v, w]] + [v, [w, u]] + [w, [u, v]] = 0.$ **7**

C) Let the Lagrangian $\alpha(q, \dot{q}) = \frac{1}{2}\dot{q}^2 - q\dot{q} + q^2$. Then show that $[p, \dot{q}^2] = -2\dot{q}$. **4**

8. A) State Modified Hamilton's principle. Derive Hamilton's equations of motion from the modified Hamilton's principle. **6**

B) For which values of α and β do the equations $Q = q^\alpha \cos\beta p$ and $P = q^\alpha \sin\beta p$ represent a canonical transformation ? **5**

C) For the Hamiltonian given by $H(q_1, q_2, p_1, p_2) = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$, where a, b are constants, show that q_1q_2 and $\frac{p_2 - bq_2}{q_1}$ are constants of motion. **5**

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M.A./M.Sc. (Semester – IV) Examination, 2011
MATHEMATICS
MT – 804 : Algebraic Topology (New)
(2008 Pattern)

Time : 3 Hours Max.Marks : 80

N.B. *: 1) Answer* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Show that there exists $f : B^n \to S^{n-1}$ with f . i = I if and only if the identity map $I : S^{n-1} \to S^{n-1}$ is homotopic to a constant map, where $i : S^{n-1} \to B^n$ is the inclusion map and '**.**' denotes the scalar product. **6**

b) Let f . g : $x \to S^n$ be continuous mappings such that f(x) $\neq$ – g(x) for all $x \in X$. Show that f $\simeq$ g. **5**

c) Let $f : S^{\perp} \to X$ be a continuous map. Show that f is null homotopic if and only if there is a continuous map $g : B^2 \to X$ with $f = g / s^{\perp}$. **5**

2. a) Prove that the relation of being of the same homotopy type is an equivalence relation. **6**

b) Let $A \subset B \subset X$. Suppose that B is a retract of X and A is a retract of B. Show that A is a retract of X. **5**

c) Show that the retract of a Housdorff space is a closed subset. **5**

3. a) Prove that a non-empty open connected subset of $\mathbb{R}^n$ is path connected. **6**

b) If f is any path then show that $f * \bar{f}$ and $\bar{f} * f$ are homotopic to null paths. **5**

c) If $\mathbb{R}^2$, let $A = \{(x, y) : x = 0, -1 \leq y \leq 1\}$ and $B = \{(x, y) : 0 < x \leq 1, y = \cos \pi / x\}$ show that $F = A \cup B$ is connected but not path connected. **5**

P.T.O.

4. a) Let x_0, $x_1 \in X$. If there is a path in X from x_0 to x_1, then show that the groups $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$ are isomorphic. **6**

   b) Prove that a contractible space is simply connected. Is converse true ? Justify your answer. **5**

   c) Let $\pi_1(X_1\ x)$ be a trivial group. If f and g are the two paths in X with $f(0) = g(0) = x$ and $f(1) = g(1)$, show that $f \sim g$. **5**

5. a) Prove that the fundamental group $\pi_1(s^{\perp})$ of the circle $s^{\perp}$ is isomorphic to the additive group $\mathbb{Z}$ of integers. **6**

   b) Show that $\mathbb{R}^{n+1} - \{0\}$ is of the same homotopy type as S^n and conclude that $\pi_1(\mathbb{R}^{n+1} - \{0\}, 0)$ is the Singleton group. **5**

   c) Let X be the punctured plane $\mathbb{R}^2 - \{(0, 0)\}$. Show that $\pi_1(x)$ is isomorphic to $\mathbb{Z}$. **5**

6. a) Prove that if X is locally connected then a continuous map $p : \tilde{X} \to X$ is a covering map if and only if for each component H of X, the map $p \mid p^{-1}(H) : p^{-1}(H) \to H$ is a covering map. **6**

   b) Prove that $\pi : \mathbb{R} \to \mathbb{R}/\mathbb{Z}$ is a covering map. **5**

   c) Let $P : \mathbb{R} \to S'$ be defined by $P(t) = e^{2\pi i f}$. Prove that P is a covering projection. **5**

7. a) Show that a fibration has unique path lifting if and only if every fiber has no non-null path. **6**

   b) Let $P : \tilde{X} \to x$ be a fibration with the unique lifting. Suppose that f and g are paths in $\tilde{X}$ with $f(0) = g(0)$ and $pf \sim pg$. Show that $f \sim g$. **5**

   c) Show that two different complexes may have same polyhedron. **5**

8. a) Prove that the closed ball $B^n (n \geq 1)$ has the fixed point property. **8**

   b) Suppose that K is a connected complex. Prove that $H_0(K)$ is isomorphic to the additive group of integers. **8**

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M.A./M.Sc. (Semester – IV) Examination, 2011
MATHEMATICS
MT – 804 : Mathematical Methods – II (Old)
(2005 Pattern)

Time : 3 Hours Max.Marks : 80

N.B. : *i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Define :
   i) Volterra integral equation of second kind
   ii) Iterated Kernels. **4**

   b) Show that the function $u(s) = \frac{1}{\left(\pi\sqrt{s}\right)}$ is a solution of the integral equation

   $\int_0^s \frac{u(t)}{\sqrt{s-t}}\,dt = 1.$ **6**

   c) Explain the method to find the solution of the integral equation

   $\phi(s) = \lambda\int_a^b K(s,t)\,\phi(t)dt$ where K(t, s) is separable Kernel. **6**

2. a) Reduce the following boundary value problem into an integral equation

   $\frac{d^2u}{ds^2} + \lambda u = 0$ with u(o) = 0, u(l) = 0. **8**

   b) Solve the homogeneous Fredholm integral equation $\phi(s) = \lambda\int_0^1 e^s e^t \phi(t)dt$. **8**

3. a) Find eigen values and eigen functions of the homogeneous Fredholm integral equation of the second kind $g(s) = \lambda\int_0^{2\pi} \sin(s+t)g(t)dt$. **8**

   b) Find the iterated Kernels for the Kernel K(s,t) = sin(s – 2t), $0 \le s, t \le 2\pi$. **8**

4. a) Solve $u(t) = 1 + \lambda\int_0^1 (1-3st)\,u(t)\,dt$ by resolvent Kernel. **8**

   b) Find the Neumann series for the solution of the integral equation

   $u(x) = 1 + x + \lambda\int_0^x K(x,t)\,u(t)dt$. **8**

5. a) Let $\psi_1(s), \psi_2(s)$, ... be a sequence of functions whose norms are all below a fixed bound M and for which the relation $\psi_n(s) - \lambda \int K(s, t)\, \psi_n(t)\, dt = 0$ holds in the sense of uniform convergence. Prove that the functions $\psi_n(s)$ form a smooth sequence of functions with finite asymptotic dimension. **8**

b) Prove that $\frac{d}{dx}\frac{\partial F}{\partial y'} - \frac{\partial F}{\partial y} = 0$ (Euler-Lagrange's equation) with usual notations. **8**

6. a) State and prove isoperimetric problem. **8**

b) Find the extremal of the functional $I = \int_{t_1}^{t_2} (x^1 y^1 + 2x^2 + 2y^2) dt$; $x^1 = \frac{dx}{dt}, y^1 = \frac{dy}{dt}$ subject to the conditions; at $t_1 = 0$, x = 0, y = 0 but the end t_2 moves on the plane $t = t_2$. **8**

7. a) Solve the symmetric integral equation $y(x) = (x+1)^2 + \int_{-1}^{1} (xt + x^2 t^2)\, y(t)\, dt$ by using Hilbert Schmidt theorem. **10**

b) State and prove Harr theorem. **6**

8. a) State and prove principle of Least action. **8**

b) Find the curve which generates a surface of revolution of minimum area which it is resolved about α-axis. **8**
